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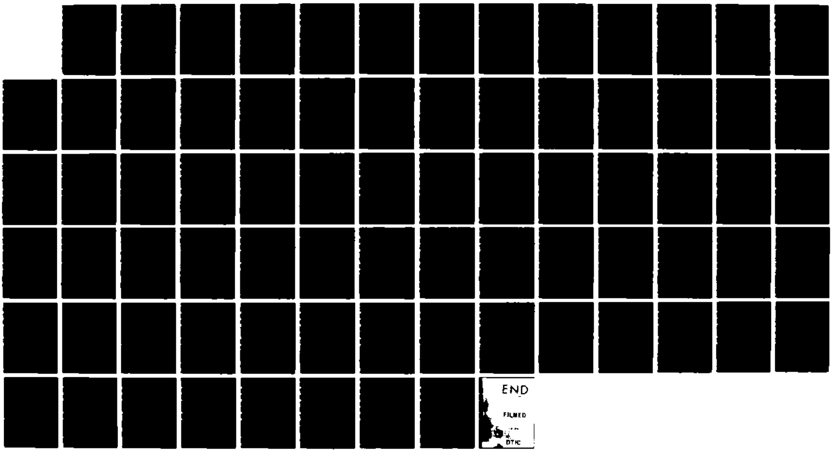

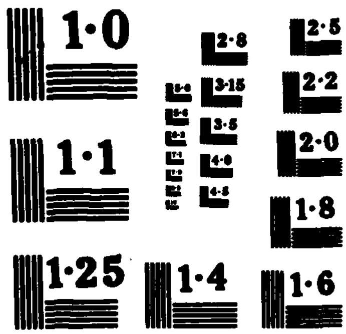
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A MATHEMATICAL THEORY OF COMMAND AND CONTROL STRUCTURES

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A MATHEMATICAL THEORY OF COMMAND AND CONTROL STRUCTURES

SUMMARY

Command control and communications (C3)

The elements of a mathematical theory for the analysis and design of organizations are presented. The focus of the research has been on information processing and decisionmaking organizations supported by C^3 systems. The mathematical framework used in modeling the individual decisionmakers, as well as the organization, is that of n-dimensional information theory. Petri Net representation of the organizational structure is used to model the interactions between organization members as well as their interactions with the C^3_u system. Comparison and evaluation of alternative organizational forms is accomplished by considering organizational performance, individual workload and the sets of satisficing *decision strategies.*

A brief description of research on distributed estimation and on information storage and flow in C^3_u systems is also included.

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LIST OF ILLUSTRATIONS

1. INTRODUCTION

The analysis of generic aspects of C^3 systems represents an area of research that requires the integration of diverse concepts and theories, if progress is to be made toward the development of a theoretical basis of their analysis and design.

The technical effort of this project was directed toward generic, long range, basic, unclassified research. The emphasis was on general methodological and technical issues, but from the perspective of the unique needs and requirements of the Air Force. The main research area was "C^3 System Structure and Organizational Forms". The goal of this effort was the development of a mathematical theory for the analysis and design of tactical military organizations supported by C^3 systems. The results of this work are presented at length in this report.

During the period of performance of this project several other tasks were undertaken in parallel with the first one, each one lasting from one to two years. Each of these tasks addressed specific aspects of the basic theoretical problems posed by the presence of C^3 systems in a distributed decisionmaking environment. They include (a) Decentralized Estimation (D.A. Castañon); (b) Information Storage and Flow in C^3 Systems (E.R. Ducot); (c) Distributed Decision Problems in Dynamic Missile Reassignment Strategies (M. Athans). Technical papers that present the significant developments in task (a) and (b) are included in appendices B and C, respectively.

A list of documents resulting from research carried under this project is included in Appendix A.

2.0 C^3 SYSTEM STRUCTURE AND ORGANIZATIONAL FORMS

2.1 Summary

Research in this area has been focused on the development of a mathematical theory for the modeling and analysis of information-processing and decisionmaking organizations. The specific organizational structures considered were motivated by tactical Air Force C^3 systems, perceived as support systems for the decisionmakers.

The first problem addressed was the development of a model of the decisionmaking process applicable to human decisionmaking in tactical situations. A model was developed using the analytical framework of n-dimensional information theory. The interactions with other decisionmakers were modeled in terms of sharing situation assessment information and issuing or receiving commands that restrict the selection of outputs or responses.

The theory developed for the single interacting decisionmaker was then extended to teams of decisionmakers forming an organization. There are three parts to the formultion of the design problem: (a) Analytic characterization of the task the organization is to perform; (b) Specification of the interactions between organization members and the environment, i.e., who receives what external inputs and who produces the organization's outputs; and (c) Specification of the interactions between organization members. These include the sharing of situation assessment information and the issuing and receiving of commands.

The theory in its current form is applicable to organizations with acyclical information structures, i.e., organizations whose digraphs depicting information flow do not contain loops. The conventional workload-performance plane for the single decisionmaker has been extended to n+1 dimensions with the n dimensions corresponding to the workload of each

one of the n members of the organization and the (n+1)st dimension to the performance measure for the organization. The theoretical development has been illustrated by designing and evaluating two three-person organizations assigned to carry an abstracted air defense task.

In the early work, the internal structure of the decisionmaking systems had been modeled as memoryless. In order to develop more realistic models in the context of the command and control process, it became necessary to introduce memory in the information theoretic framework so that inputs that are statistically dependent could be considered.

Three types of storage have been modeled: buffer storage, temporary memory and permanent memory. Since this analysis addressed the processing of sequential inputs that are dependent on each other, information rates and the Partition Law for Information Rates were used. Consequently, inputs were modeled by discrete stationary ergodic sources. The model of permanent memory was then used to analyze a typical situation for a decisionmaker: the processing of two distinct tasks in parallel -- the dual task problem.

The model of the decisionmaking process contains a set of algorithms in the situation assessment stage and another set in the response selection stage. For the early version of the model, the simplifying assumption was made that the algorithms were deterministic. The theory has since been extended to include stochastic algorithms. As expected, it was shown that the presence of non-deterministic algorithms increases the total activity by increasing the component that corresponds to internally generated information or noise.

The modeling of preprocessors of various types was also considered. The objective was to develop an analytical formulation of the problem of introducing decision aids in an organization. Both deterministic and stochastic preprocessors have been studied and their effect on workload and performance has been investigated.

Finally, an additional task was introduced, namely, the modeling of the organizations using the Petri Net formalism so that the organizational delay could be evaluated. A system matrix for synchronous protocols was defined and an algorithm for calculating delays was implemented and tested on the two three-person organizations.

A comprehensive review of the approach and the results is presented in the following sections.

2.2 The Design Problem

In considering organizational structures for teams of decisionmakers, a designer must address the questions of who receives what information and who is assigned to make which decisions. The resolution of these questions specifies the organizational form. The designer's problem is the selection of a form so that the resulting organization meets its performance specifications and the individual members are not overloaded, i.e., the task requirements do not exceed their individual processing limitations.

While the role of the human decisionmakers is central to the design problem, the latter cannot be decoupled from the consideration of the information system that supports the organization. Consider, for example, a tactical military organization supported by a command, control, and communications (C^3) system. Information is collected from many sources, distributed to appropriate units in the organization for processing, and used by the commanders and their staff to make decisions. These decisions are then passed to the units responsible for carrying them out. Thus, a given organization design implies the existence of a C^3 system that supports it. Conversely, the presence of a C^3 system in support of an organization modifies the latter's operations; it may create operational modes not foreseen during the organizational design phase. Therefore, if a quantitative description of the organization design problem is to be developed, it must take into account not only the organization members, but

also the collection of equipment and procedures that constitute the organization's C^3 system.

In order to develop a quantitative methodology for the analysis and evaluation of information processing and decisionmaking organizations, it is necessary that a set of compatible models be obtained that describe the organization and its environment. This modeling effort has been divided in three steps. The first one is the modeling of the tasks the organization is to execute and the definition of the boundary between the organization and its environment. The second step is the selection of mathematical models that describe the members of the organization. The third step is the modeling of organizational form, i.e., the specification of the information and decision structures that characterize the organization. This step includes the specification of the protocols for information exchange and the modeling of the communication systems, the data bases, and the decision aids that the organization uses to perform its tasks.

The methodology itself consists of two main parts. In the first one, the <u>analysis of the organization</u>, the models are used to describe the organization in terms of a locus defined on a generalized performance - workload space. This locus is obtained by computing an index of performance for the organization and measures of the workload for each individual member of the organization as functions of the admissible decision strategies used by the decisionmakers. The second part of the methodology addresses the question of <u>evaluating organizational designs</u> and comparing alternative structures.

The analytical framework used for modeling the tasks, the individual organization members, the C^3 system, and the organization as a whole is that of n-dimensional information theory [1]. A brief description of the key quantities and of the partition law of information [2] is presented in the next section.

2.3 Information Theoretic Framework

Information theory was first developed as an application in communication theory [3]. But, as Khinchin [4] showed, it is also a valid mathematical theory in its own right, and it is useful for applications in many disciplines, including the modeling of simple human decisionmaking processes [5] and the analysis of information-processing systems.

There are two quantities of primary interest in information theory. The first of these is <u>entropy</u>: given a variable x, which is an element of the alphabet X, and occurs with probability p(x), the entropy of x, H(x), is defined to be

$$H(x) = \sum_{x} p(x) \log p(x) \tag{2.1}$$

and is measured in bits when the base of the algorithm is two. The other quantity of interest is <u>average mutual information</u> or <u>transmission</u>: given two variables x and y, elements of the alphabets X and Y, and given p(x), p(y), and p(x|y) (the conditional probability of x, given the value of y), the transmission between x and y, T(x:y) is defined to be

$$T(x:y) = H(x) - H_y(x) \tag{2.2}$$

where

$$H_y(x) = - \sum_{y} p(y) \sum_{x} p(x|y) \log p(x|y) \tag{2.3}$$

is the conditional uncertainty in the variable x, given full knowledge of the value of the variable y.

McGill [1] generalized this basic two-variable input-output theory to N dimensions by extending Eq. (2.2):

$$T(x_1:x_2:\ldots:x_N) = \sum_{i=1}^{N} H(x_i) - H(x_1,x_2,\ldots,x_N) \tag{2.4}$$

For the modeling of memory and of sequential inputs which are dependent on each other, the use of the entropy rate, $\bar{H}(x)$, which describes the average entropy of x per unit time, is appropriate:

$$\bar{H}(x) = \lim_{m\to\infty} \frac{1}{m} H[x(t),\ x(t+1),\ldots,x(t+m-1)] \tag{2.5}$$

Transmission rates, $\bar{T}(x:y)$, are defined exactly like transmission, but using entropy rates in the definition rather than entropies.

The Partition Law of Information [2] is defined for a system with N-1 internal variables, w_1 through w_{N-1}, and an output variable, y, also called w_N. The law states

$$\sum_{i=1}^{N} H(w_i) = T(x:y) + T_y(x:w_1,w_2,\ldots,w_{N-1}) + T(w_1:w_2:\ldots:w_{N-1}:y)$$

$$+ H_x(w_1,w_2,\ldots,w_{N-1},y) \tag{2.6}$$

and is easily derived using information theoretic identities. The left-hand side of (2.6) refers to the total activity of the system, also designated by G. Each of the quantities on the right-hand side has its own interpretation. The first term, T(x:y), is called <u>throughput</u> and is designated G_t. It measures the amount by which the output of the system is related to the input. The second quantity,

$$T_y(x:w_1,w_2,\ldots,w_{N-1}) = T(x:w_1,w_2,\ldots,w_{n-1},y) - T(x:y) \tag{2.7}$$

is called blockage and is designated G_b. Blockage may be thought of as the amount of information in the input to the system that is not included in the output. The third term, $T(w_1:w_2:\ldots:w_{N-1}:y)$ is called coordination and designated G_c. It is the N-dimensional transmission of the system, i.e., the amount by which all of the internal variables in the system constrain each other. The last term, $H_x(w_1,w_2,\ldots,w_{N-1},y)$, designated by G_n represents the uncertainty that remains in the system variables when the input is completely known. This noise should not be construed to be necessarily undesirable, as it is in communication theory: it may also be thought of as internally-generated information supplied by the system to supplement the input and facilitate the decisionmaking process. The partition law may be abbreviated:

$$G = G_t + G_b + G_c + G_n \tag{2.8}$$

A statement completely analogous to (2.8) can be made about information rates by substituting entropy rate and transmission rates in (2.6).

2.4 Task Model [6,7]

The organization, perceived as an open system [8], interacts with its environment; it receives signals or messages in various forms that contain information relevant to the organization's tasks. These messages must be identified, analyzed, and transmitted to their appropriate destinations within the organization. From this perspective, the organization acts as an information user.

Let the organization receive data from one or more sources external to it. Every τ_n units of time on the average, each source n generates symbols,

signals, or messages x_{ni} from its associated alphabet X_n, with probability p_{ni}, i.e.,

$$p_{ni} = p(x_n = x_{ni}) \quad ; \quad x_{ni} \in X_n \qquad i = 1,2,,\ldots\gamma_n \tag{2.9}$$

$$\sum_{i=1}^{\gamma_n} p_{ni} = 1 \quad ; \quad n = 1,2,\ldots,N' \tag{2.10}$$

where γ_n is the dimension of x_n. Therefore, $1/\tau_n$ is the mean frequency of symbol generation from source n.

The organization's task is defined as the processing of the input symbols x_n to produce output symbols. This definition implies that the organization designer knows a priori the set of desired responses Y and, furthermore, has a function or table $L(x_n)$ that associates a desired response or a set of desired responses, elements of Y, to each input $x_n \in X_n$.

It is assumed that a specific complex task that must be performed can be modeled by N' sources of data. Rather than considering these sources separately, one supersource composed of these N' sources is created. The input symbol $\underline{x}'$ may be represented by an N'-dimensional vector with each of the sources represented by a component of this vector, i.e.,

$$\underline{x}' = (x_1, x_2, \ldots, x_N) \quad ; \quad \underline{x}' \in X \tag{2.11}$$

To determine the probability that symbol $\underline{x}_j'$ is generated, the independence between components must be considered. If all components are mutually independent, then p_j is the product of the probabilities that each component of $\underline{x}_j'$ takes on its respective value from its associated alphabet:

$$p_j = \prod_{n=1}^{N'} p_{nj} \tag{2.12}$$

If two or more components are probabilistically dependent on each other, but as a group are mutually independent from all other components of the input vector, then these dependent components can be treated as one supercomponent, with a new alphabet. Then a new input vector, $\underline{x}$, is defined, composed of the mutually independent components and these supercomponents.

This model of the sources implies synchronization between the generation of the individual source elements so that they may, in fact, be treated as one input symbol. Specifically, it is assumed that the mean interarrival time for each component τ_n is equal to τ. It is also assumed that the generation of a particular input vector, $\underline{x}_j$, is independent of the symbols generated prior to or after it.

The last assumption can be weakened, if the source is a discrete stationary ergodic one with constant interarrival time τ that could be approximated by a Markov source. Then the information theoretic framework can be retained [6].

The vector output of the source is partitioned into groups of components that are assigned to different organization members. The j-th partition is denoted by $\underline{x}^j$ and is derived from the corresponding partition matrix $\underline{\pi}^j$ which has dimension $n_j \times N$ and rank n_j, i.e.,

$$\underline{x}^j = \underline{\pi}^j \underline{x}. \tag{2.13}$$

Each column of $\underline{\pi}^j$ has at most one non-zero element. The resulting vectors $\underline{x}^j$ may have some, all, or no components in common.

The set of partitioning matrices $\{\underline{\pi}^1, \underline{\pi}^2, \ldots, \underline{\pi}^n\}$ shown in Figure 1

specify the components of the input vector received by each member of the subset of decisionmakers that interact directly with the organization's environment. These assignments can be time invariant or time varying. In the latter case, the partition matrix can be expressed as

$$\pi^j(t) = \begin{cases} \pi_o^j & \text{for } t \in T \\ 0 & \text{for } t \notin T \end{cases} \tag{2.14}$$

The times at which a decisionmaker receives inputs for processing can be obtained either through a deterministic (e.g., periodic) or a stochastic rule. The question of how to select the set of partition matrices, i.e., design the information structure between the environment and the organization, has been addressed by Stabile [7,9].

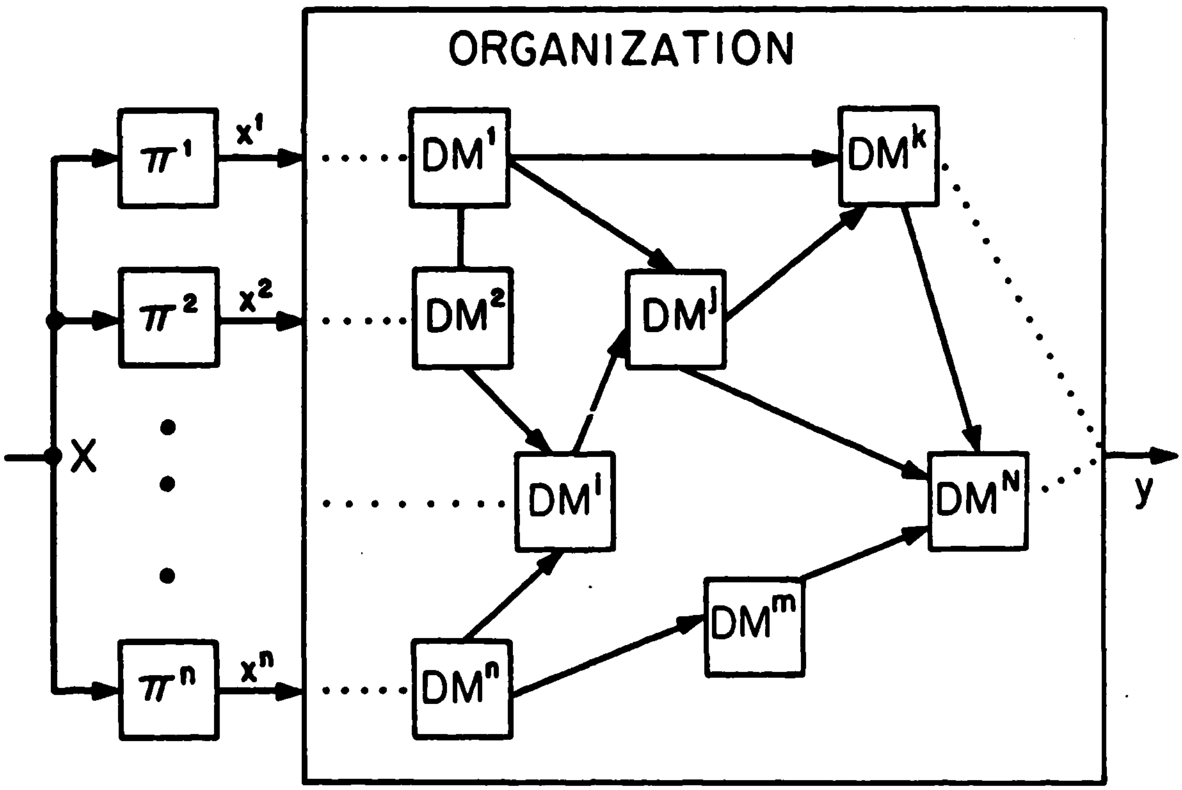

Figure 1. Information Structures for Organizations

2.5 Model of the Organization Member [10,11,12]

The complete realization of the model of the decisionmaker (DM) who is interacting with other organization members and with the environment is shown schematically in Figure 2.

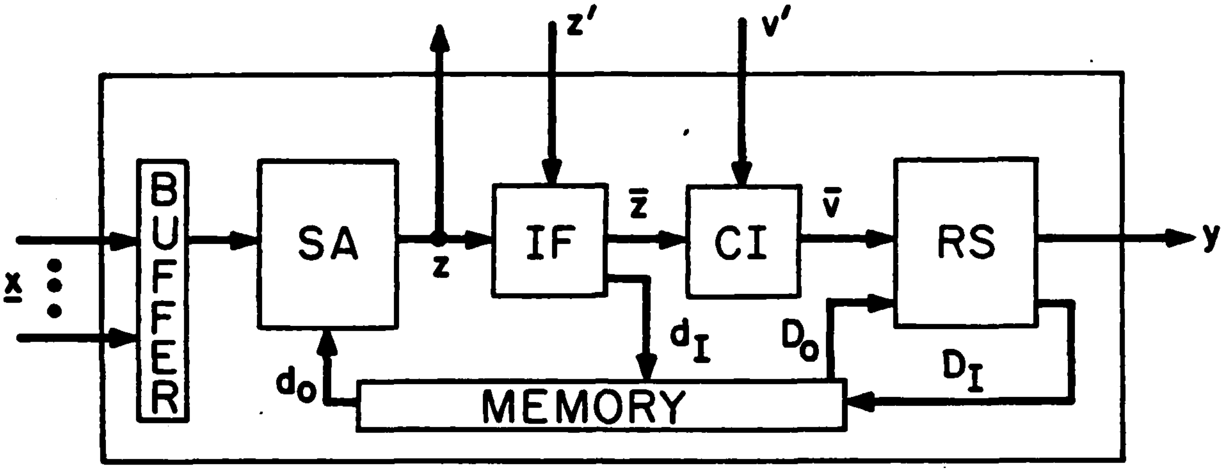

Figure 2. The Interacting Decisionmaker with Memory

The DM receives singals $\underline{x} \in X$ from the environment with interarrival time τ. A string of signals may be stored first in a buffer so that they can be processed together in the situation assessment (SA) stage. The SA stage contains algorithms that process the incoming signals to obtain the assessed situation $\underline{z}$. The SA stage may access the memory or internal data base to obtain a set of values d_o. The assessed situation z may be shared with other organization members; concurrently, the DM may receive the supplementary situation assessment z′ from other parts of the organization; the two sets z and z′ are combined in the information fusion (IF) processing stage to obtain $\bar{z}$. Some of the data (d_I) from the IF process may be stored in memory.

The possibility of receiving commands from other organization members is modeled by the variable v′ and a command interpretation (CI) stage of processing is necessary to combine the situation assessment $\bar{z}$ and v′ to

arrive at the choice $\bar{v}$ of the appropriate strategy to use in the response selection (RS) stage. The RS stage contains algorithms that produce outputs y in response to the situation assessment $\bar{z}$ and the command inputs. The RS stage may access data from or store data in memory [6,13].

A more detailed description of the decisionmaker model without buffer or memory is shown in Figure 3. This figure shows the internal structure of the four processing stages: SA, IF, CI, and RS. The situation assessment stage consists of a set of U algorithms (deterministic or not) that are capable of producing some situation assessment z. The choice of algorithms is achieved through specification of the internal variable u in accordance with the situation assessment strategy p(u) or p(u|x), if a decision aid (e.g., a preprocessor) is present. A second internal decision is the selection of the algorithm in the RS stage according to the response selection strategy $p(\bar{v}|\bar{z},v')$. The two strategies, when taken together, constitute the internal decision strategy of the decisionmaker.

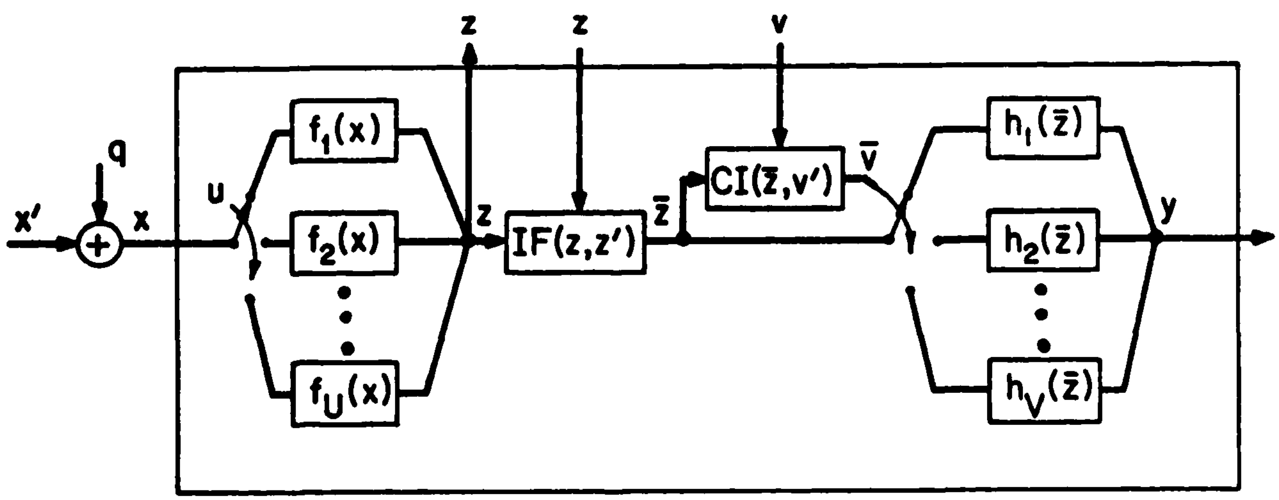

Figure 3. Detailed Model of the Interacting Decisionmaker

The analytical framework presented in Section 2.3, when applied to the single interacting decisionmaker with deterministic algorithms in the SA and RS stages, yields the four aggregate quantities that characterize the

information processing and decisionmaking activity within the DM [10,12]:

Throughput:

$$G_t = T(x,z',v':z,y) \tag{2.15}$$

Blockage:

$$G_b = H(x,z',v') - G_t \tag{2.16}$$

Internally generated information:

$$G_n = H(u) - H_{\bar{Z}}(v) \tag{2.18}$$

Coordination:

$$G_c = \sum_{i=1}^{U} p_i g_c^i(p(x)) + \alpha \ \ H(p \ ) + H(z) + g_c^{IF}(p(z,z')) + g_c^{CI}(p(\bar{z},v')$$

$$+ \sum_{j=1}^{V} p_j g_c^i(p(\bar{Z}|\bar{V} = j)) + \alpha_j \ H(p_j) + H(y)$$

$$+ H(z) + H(\bar{Z}) + H(\bar{Z},\bar{V}) + T_z(x':z') + T_{\bar{Z}}(x',z':v') \tag{2.18}$$

The expression for G_n shows that it depends on the two internal strategies $p(u)$ and $p(v|\bar{Z})$ even though a command input may exist. This implies that the command input v' modifies the DM's internal decision after $p(v|\bar{Z})$ has been determined.

In the expressions defining the system coordination, p_i is the probability that algorithm f_i has been selected for processing the input x

and p_j is the probability that algorithm h_j has been selected, i.e., u = i and $\bar{v}$ = j. The quantities g_c represent the internal coordinations of the corresponding algorithms and depend on the distribution of their respective inputs; the quantities α_i, α_j are the number of internal variables of the algorithms f_i and h_j, respectively. Finally, the quantity H is the entropy of a binary random variable:

$$H(p) = - p\log_2 p - (1 - p)\log_2(1-p) \qquad (2.19)$$

Equations (2.15) to (2.18) determine the total activity G of the decisionmaker according to the partition law of information (2.6). The activity G can be evaluated alternatively as the sum of the marginal uncertainties of each system variable. For any given internal decision strateg;, G and its component parts can be computed.

Since the quantity G may be interpreted as the total information processing activity of the system, it can serve as a measure of the workload of the organization member in carrying out his decisionmaking task.

The qualitative notion that the rationality of a human decisionmaker is not perfect, but is bounded [14], has been modeled as a constraint on the total activity G:

$$G = G_t + G_b + G_n + G_c \leq F \tau_o \qquad (2.20)$$

where τ_o is the symbol interarrival time and F is the maximum rate of information processing that characterizes a decisionmaker. This constraint implies that the decisionmaker must process his input at a rate that is least equal to the rate with which they arrive. For a detailed discussion of this particular model of bounded rationality, see Boettcher and Levis [10].

Weakening the assumption that the algorithms are deterministic, changes

the numerical values of G_n and of the coordination term G_c [15]. If memory is present in the model, then additional terms appear in the expressions for the coordination rate and for the internally generated information rate [6,13].

2.6 Organizational Form

In order to define an organizational structure, the interactions between the human decisionmakers that constitute the organization must be specified. The interactions between DMs and the environment have already been described in Section 2.4. The internal interactions between DMs consist of receiving inputs from other DM's, sharing situation assessments, receiving command inputs, and producing outputs that are either inputs or commands to other DM's. The detailed specification of the interactions requires the determination of what information is to be passed among individual organization members and the precise sequence of processing events, i.e., the standard operating procedure or communication and execution protocol of the organization.

Information structures that can be modeled within this analytical framework are those that represent synchronized, acyclical information flows. Since inputs are assumed to arrive at a fixed average rate, the organization is constrained to produce outputs at the same average rate. The overall response is made up, in general, of the responses of several members; therefore, each member is assumed to complete the processing corresponding to a particular input at the same average rate.

Within this overal rate synchronization, however, processing of a specific input symbol or vector takes place in an asynchronous manner. If the requisite inputs for a particular stage of processing are present, then processing can begin without regard to any other stage, which implies that concurrent processing is present. For example, as soon as the organization input arrives and is partitioned through π, processing of x begins to obtain z. The IF stage must wait, however, until both the z and z' values are

present. Each stage of processing is thus event-driven; a well-defined sequence of events is therefore an essential element of the model specification.

Acyclical information structures are those whose directed graphs representing the flows of information do not contain any cycles or loops. This restriction is made to avoid deadlock and circulation of messages within the organization. Deadlock occurs when one DM is waiting for a message from another in order to proceed with his task, while the second one is in turn waiting for an input from the first.

The system theoretic representation of the organizational form is useful for showing the various processing stages or subsystems. For example, in Figure 4, a three person organization is shown in block diagram form in which the first and third members send information to the second, who in turn can issue commands to the other two DMs.

Evaluation of the various information theoretic quantities, including total activity, can be accomplished readily, using the decomposition property of the information theoretic framework [2]. However, the internal information structure of the organization is often ambiguous when represented in block diagram terms. For example, the requirement that both z^{12} and z^{32} be present before the second DM can fuse the information is not apparent from Figure 4. An alternate representation is needed which shows explicitly the information structure without compromising the usefulness of the information theoretic decomposition property.

Petri Nets [16] and the data flow-schema [17,18] have been developed as models of information flow for systems with asynchronous, concurrent processing activities. Three basic elements are used in their structure: places, transitions, and directed arcs which connect the two. Places and transitions represent conditions and events, respectively. No event occurs unless the requisite conditions are met, but the occurrence of an event gives rise to new conditions. Tokens are used to mark which conditions are

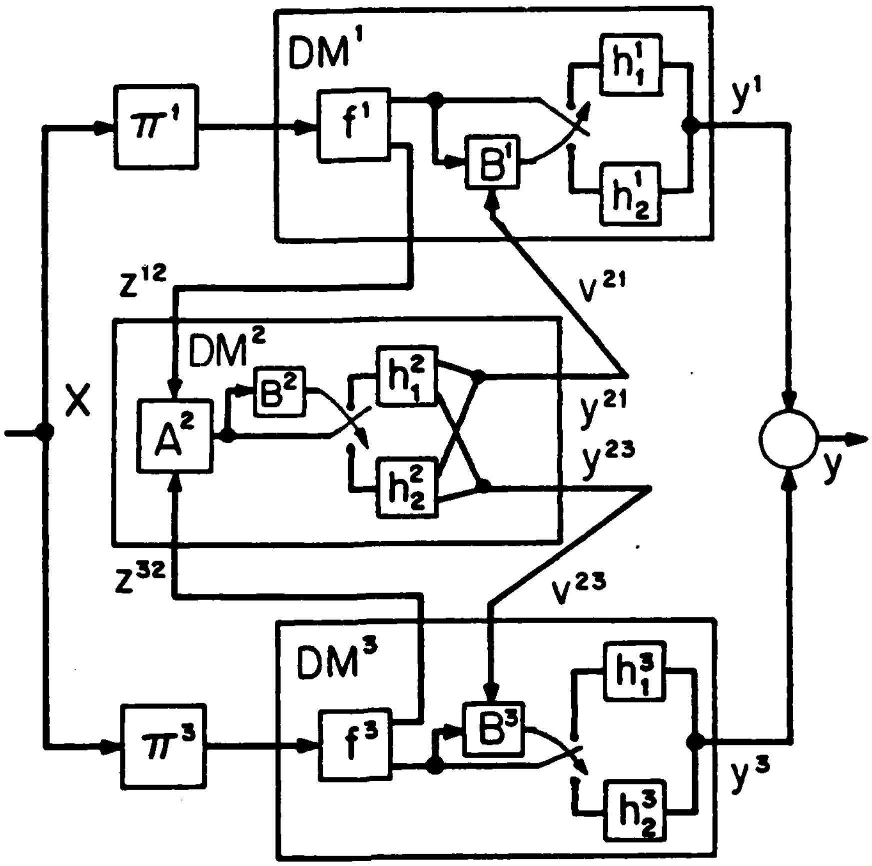

Figure 4. Block diagram representation of three person organization

in effect; when all input places to (conditions for) a transition contain a token (are satisfied), then the event can occur, which in turn results in the generation of tokens for output places. Since tokens are carriers of data, each transition is a processor which generates a result from the input data and deposits it on an output token which then along a directed arc to the next stage of processing.

To represent the information theoretic decisionmaking model using the Petri Net formalism, a simple translation in structure is made: distinct inputs and outputs of each subsystem are assigned places and the processing within a subsystem is represented by a transition [19]. Associated with each transition is the set of internal variables of the subsystem, exclusive of the input variables, which are accounted for separately by the input places. By assuming a probability distribution on the organization's

inputs, distributions are also included on the places in the structure. Therefore, distributions are also present on subsystem variables, and all information theoretic quantities are well-defined and can be computed as before.

The organization structure shown in Figure 4, represented as a Petri Net is shown in Figure 5. In addition to places, transitions, and directed arcs, the structure contains a new element, the switch. This is a logical element which direct the flow of tokens. The switch may take values independently, or the value is determined as a result of the processing by algorithm B contained in the command input block. Since the structure shown in Figure 5 is equivalent to the system theoretic structure in Figure 4, the internal variable definition and all information theoretic quantities remain unchanged. However, the information structure of the organization is made explicit in Figure 5.

Once an input X is partitioned, the processing by each DM in his respective SA stage (algorithms f) begins concurrently and asynchronously. This is evident from the representation. Note that because of the assumed synchronization with respect to organization inputs, there can be at most one data token in any single place. The structure is obviously acyclical and deadlock in the organization is prevented.

The Petri Net representation leads directly to the representation of delays in each stage of processing in terms of an array [19]. Each decisionmaker is represented in the array be a sequence of delays where each delay represents the time it takes for a message or task to be processed by a given subsystem of the DM. Each such subsystem appears as a transition in the Petri Net representation of the DM. The array entry also indicates the origin of the messages to be processed as well as their destination. The presence of switches (decisions) is also shown. An algorithm has been designed for the calculation of delays for any origin-destination pair. This algorithm is suitable for implementation on microprocessor based computers.

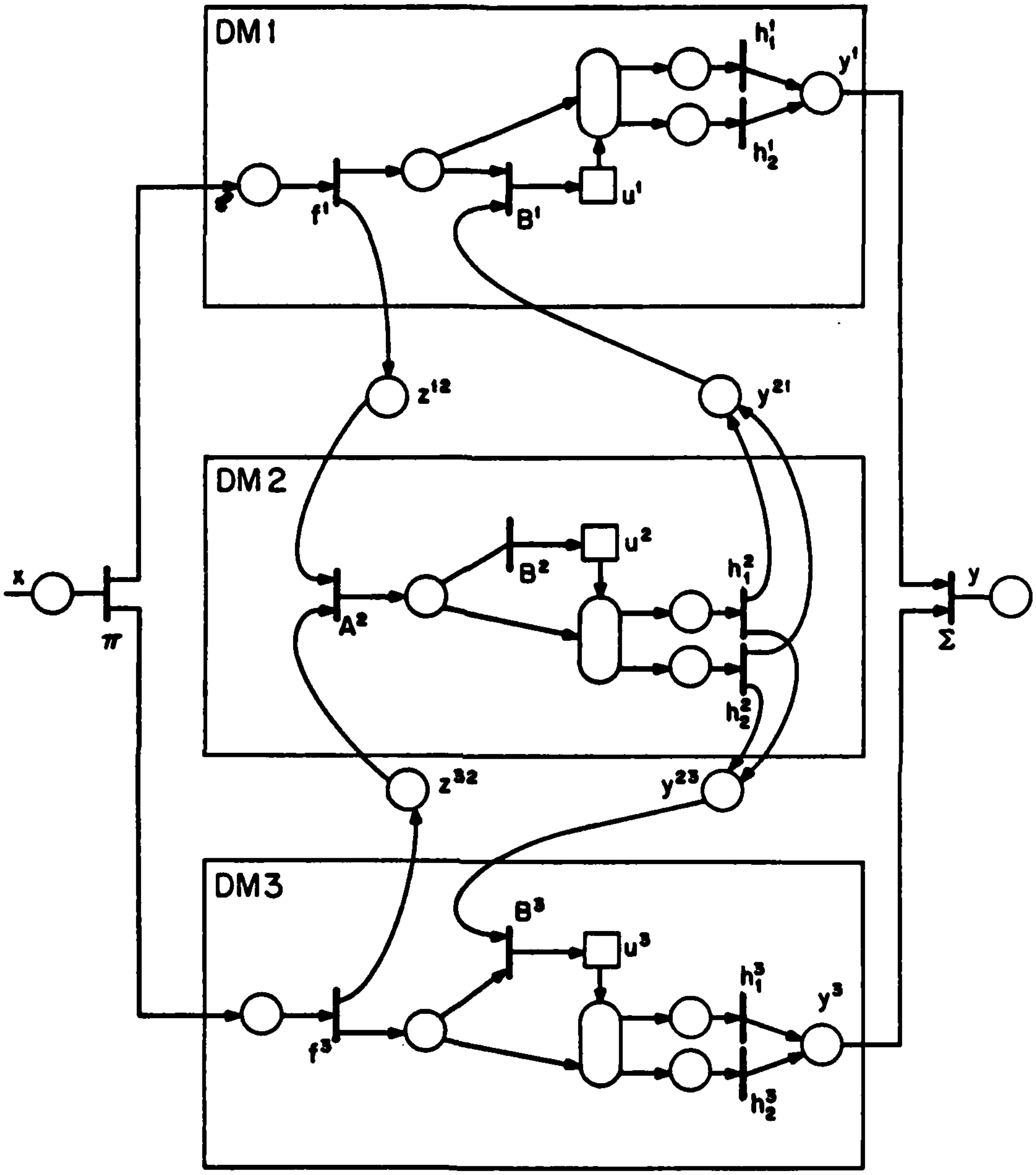

Figure 5. Data-flow representation of three person organization structure.

The Petri Net or data flow framework provides a natural way for describing in a precise manner the interactions between the DM's and the data bases and decision aids present in the organization. The presence of

data bases, an integral part of a C^3 system, requires the introduction of two additional modeling elements. The first is the query-response process. The second is the modeling of the data storage devices themselves. Consider, for example, the situation assessment subsystem shown in Figure 6. In accordance with the internal strategy u, an algorithm is chosen to process the input x. However, this algorithm may require parameters (e.g., terrain information, meteorological data) or past situation assessments in order to do the processing. The data base is accessed and queried for this information through the signal D_I. The data from the data base are provided to the SA subsystem of the DM through D_O. The same link, DI, can be used to update the information in the data base. Clearly, the block diagram representation is ambiguous; the data flow formalism allows for the precise modeling of the fact that data are requested only when certain conditions are met.

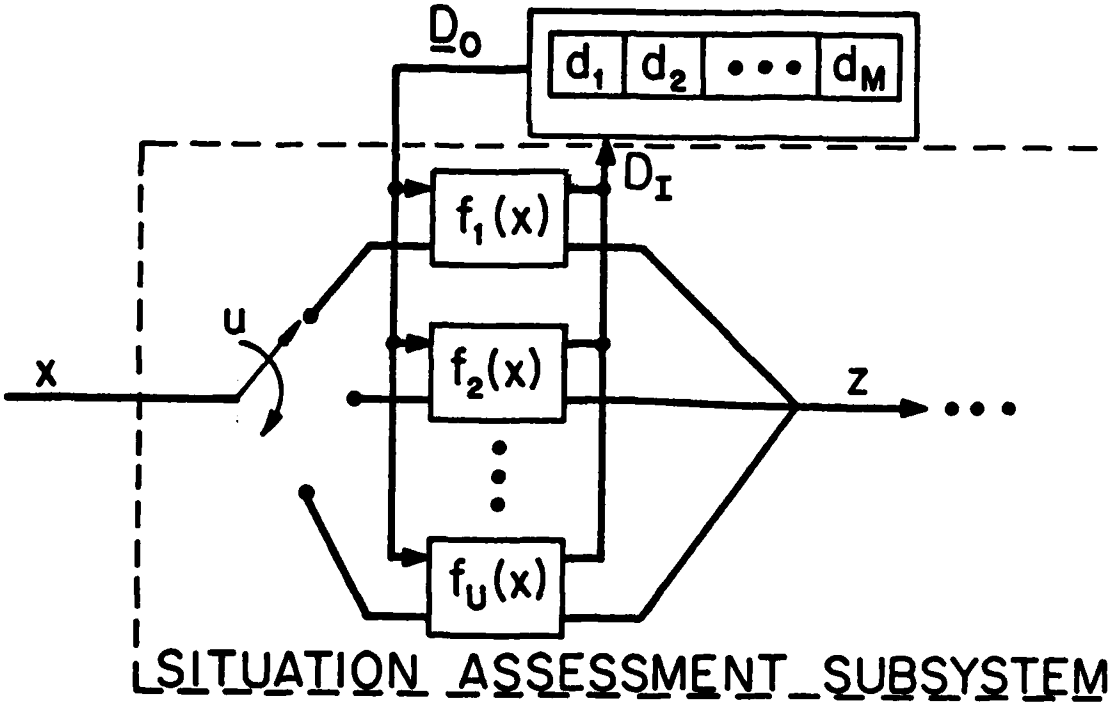

Figure 6. Model of SA subsystem with data base access

Consider next the effect of a data base containing data that do not change durg the execution of a task, i.e., the data are fixed. At first

glance, it might seem that the addition of the data base with fixed values would have no effect on the total information theoretic rate of activity of the system, i.e.,

$$\bar{H}(d_i) = 0 \qquad i = 1,2,\ldots,M \tag{2.21}$$

However, the problemn is more complex. For example, if each algorithm f_i accesses β_i parameter values from the data base (in contrast to having these values fixed within the algorithm itself) then the rates of throughput, blockage, and noise of the combined system will not be affected, but the coordination term will have additional activity rate:

$$\Delta\bar{G}_c = \sum_{i=1}^{U} \beta_i \, H[p(u=i)] \tag{2.22}$$

Since a data base increases the overall activity of the system without creating any change in its input-output characteristic, one would question its presence. There are several advantages: (a) reduction in the information that needs to exist within the algorithms or within the decisionmaker model, (b) increased flexibility in the use of algorithms and hence possible reduction in the number of algorithms, and (c) access to common data by several organization members. Even though there is increased coordination activity due to the interaction between the DM and the data base, the total activity of the DM may be reduced — the task may be redesigned to fall within the bounded rationality constraints.

Similar arguments apply to the modeling and design of decision aids. Preprocessors, a class of decision aids, are assumed to be located between a source (whether within the organization or external to it) and a decisionmaker. The basic preprocessor model that has been analyzed [15] employs a matching algorithm to partition the input alphabet so that the appropriate decision strategy can be used in the situation assessment stage. The preprocessor allows the use of internal strategies of the fcrm $p(u|x)$ in place of $p(u)$ in the SA stage. The ability of the preprocessor in reducing

the workload of a decisionmaker was found to depend on the input uncertainty, the decision strategies and the algorithms within the preprocessor. A preprocessor that can do filtering of input data was shown to upgrade the performance of a DM with bounded rationality by withholding irrelevant inputs and, therefore, reducing the input rate to the DM. As a result, more time became available to the decisionmaker to perform the relevant tasks.

An approach to modeling the organizational form — the specification of the structure and of protocols for interaction between DM's and the supporting command, control, and communication system — has been presented. It is based on an integration of Petri Nets with the information theoretic framework used in the quantitative modeling of the decisionmaking process. This framework allows for the explicit representation of a wide variety of organizational structures.

2.7 Analysis of Organizations

As stated in Section 2.4, it is assumed that the designer knows a priori the set of desired responses Y to the input set X. Then the performance of the organization in accomplishing its tasks can be evaluated using the approach shown in Figure 7.

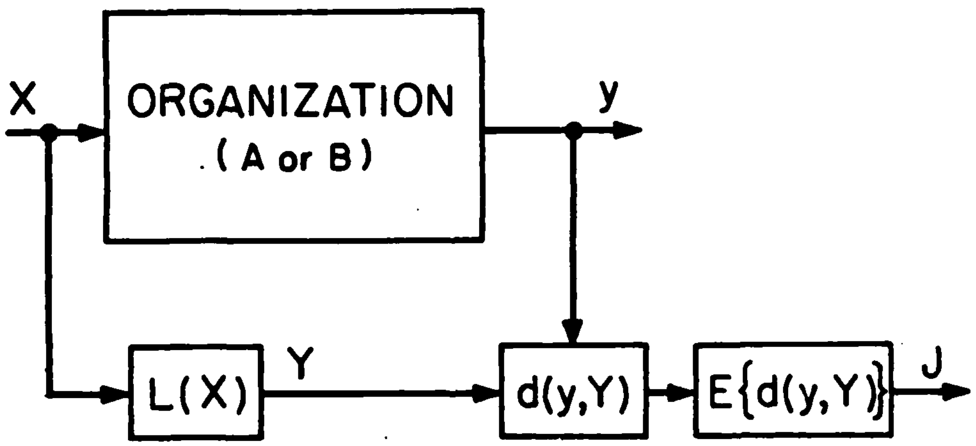

Figure 7. Performance evaluation of an organization

The organization's actual response y can be compared to the desired set Y_d and a cost assigned using a cost function $d(y,Y)$. The expected value of the cost, obtained by averaging over all possible inputs, can serve as a performance index, J, for the organization. For example, if the function $d(y,Y)$ takes the value of zero when the actual response is one of the desired ones and unity otherwise, then

$$J = E\{d(y,Y)\} = p(y \neq Y_d) \tag{2.23}$$

In this case, J represents the probability of the organization making the wrong decision, i.e., the probability of error. Once the organizational form is specified, the total processing activity G and the value of organizational performance J can be expressed as functions of the internal decision strategies selected by each decisionmaker. Let an internal strategy for a given decisionmaker be defined as pure, if both the situation assessment strategy $p(u)$ and the response selection strategy $p(v|\bar{z})$ are pure, i.e., an algorithm f_i is selected with probability one and an algorithm h_j is selected also with probability one when the situation assessed as being $\hat{z}$:

$$D_k = \{p(u=i) = 1 \quad ; \quad p(v=j|\bar{z}=\hat{z}) = 1\} \tag{2.24}$$

for some i, some j, and for each $\hat{z}$ element of the alphabet $\bar{Z}$. There are n possible pure internal strategies,

$$n = U \cdot V^M \tag{2.25}$$

where U is the number of f algorithms in the SA stage, V the number of h algorithm in the RS stage and M the dimension of the set $\bar{Z}$. All other internal strategies are mixed [20] and are obtained as convex combinations of pure strategies:

$$D(p_k) = \sum_{k=1}^{n} p_k D_k \tag{2.26}$$

where the weighting coefficients are probabilities.

Corresponding to each $D(p_k)$ is a point in the simplex

$$\sum_{k=1}^{n} p_k = 1, \quad p_k \geq 0 \quad \forall k \tag{2.27}$$

The possible strategies for an individual DM are elements of a closed convex hyperpolyhedron of dimension n-1 whose vertices are the unit vectors corresponding to pure strategies.

Because of the possible interactions among organization members, the value of G depends not only on $D(p_k)$ but also on the internal decisions of the other decisionmakers. A pure organizational strategy is defined as a M-tuplet of pure strategies, one from each DM:

$$\Delta_{1,2,\ldots,M} = \{D_{k_1}, D_{k_2}, \ldots, D_{k_M}\} \tag{2.28}$$

Independent internal decision strategies for each DM, whether pure or mixed, induce a behavioral strategy [20] for the organization, which can be expressed as

$$\Delta = \sum_{1,2,\ldots,M} (\Delta_{1,2,\ldots,M} \prod_{i=1}^{M} p_{k_i}) \tag{2.29}$$

where p_k is the probability of using pure strategy, D_k. Because each DM is assumed to select his strategy independently of other DM's, the strategy

space of the organization, S^O, is determined as the direct sum of the individual DM strategy spaces:

$$S^O = S^1 \oplus S^2 \oplus \ldots \oplus S^M \tag{2.30}$$

where S^i denotes the individual DM strategy space. The dimension of S^O is given by

$$s = \dim S^O = \sum_{i=1}^{M} (n_i - 1)$$

Thus, the organizational strategies are elements of an s-dimensional closed convex hyperpolyhedron.

As Δ ranges over S^O, the corresponding values of the performance index J and the activity or workload of each individual organization member can be computed. In this manner, the set S^O is mapped into a locus on the M+1 dimensional performance-workload space, namely the space $(J, G^1, G^2, \ldots, G^M)$. Note that only the internal processing activity of the decisionmakers is presented in the locus and not the total activity of the system which includes the activity of the decision aids, data bases, and other components of the supporting C^3 system. Consequently, the bounded rationality constraints become hyperplanes in the performance-workload space. Since the bounded rationality constraint for all DM's depends on τ, the admissible internal decision strategies of each DM will also depend on the tempo of operations. The unconstrained case can be thought of as the limiting case when $\tau \rightarrow \infty$.

The methodology for the analysis of organizational structures allows for the formulation and solution of two problems: (a) the determination of the organizational strategies that minimize J and (b) the determination of the set of strategies for which $J \leq \bar{J}$. The first problem is one of

optimization while the latter is formulated so as to obtain satisficing strategies with respect to a performance threshold $\bar{J}$. The satisficing condition also defines a plane in the performance-workload space that is normal to the J axis and intersects it at $\bar{J}$. All points on the locus on or below this plane which also satisfy the bounded rationality constraint for each decisionmaker in the organization define the set of satisficing decision strategies. Analytical properties of this locus as well as a computational approach to its efficient construction have been discussed in [10,11,12].

A qualitative evaluation of an organizational structure can be made by comparing the performance-workload locus to the space defined by the satisficing and bounded rationality constraints. In the same manner, alternative organizational strutures can be compared by considering their respective loci.

Since individual decisionmakers select their own decision strategies independently of all other organization members, a particular organizational form can yield a broad range of performance as illustrated by the locus in the performance-workload space. The designer must assess, therefore, the likelihood that strategies which lead to satisficing performance will be selected. A possible measure of this consistency between individually selected strategies can be obtained by comparing the locus of the satisficing strategies to the locus of the organization's strategy space S^o. Let R^i be the subspaces of organization strategies which are feasible with respect to the bounded rationality constraint of each DM, i.e.,

$$R^i = \{\Delta \mid G^i(\Delta) \leq F^i \tau\} \tag{2.31}$$

and let R^J contain the strategies that satisfy the performance threshold $\bar{J}$:

$$R^J = \{\Delta \mid J(\Delta) \leq \bar{J}\} \tag{2.32}$$

The subspace of satisficing strategies R^O is given by:

$$R^O = R^1 \cap R^2 \cap \ldots \ R^M \cap R^J \tag{2.33}$$

The volume of R^O, denoted by $V(R^O)$ is compared with that of S^O, $V(S^O)$, to determine the measure of consistency, Q, i.e.,

$$Q = V(R^O)/V(S^O) \tag{2.34}$$

The ratio Q is a monotonic function of $\bar{J}$ and τ with minimum zero and maximum one. A null value for Q implies that no combination of strategies of the individual decisionmakers will satisfy the design specifications, while unity implies that all organizational strategies are feasible, i.e., satisfy the bounded rationality constraints and the performance specifications.

Since Q can be expressed as a function of $\bar{J}$ and τ only, it can be plotted in the three-dimensional space $(Q,\bar{J},\tau)$. A typical plot from a three DM example [12] is shown in Figure 8.

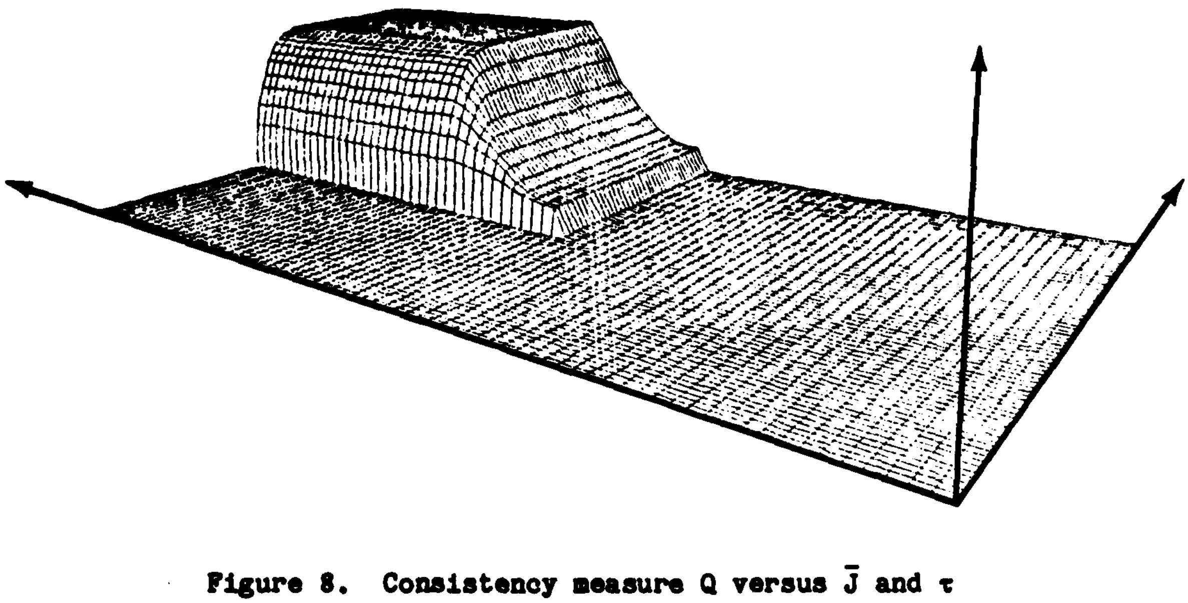

Figure 8. Consistency measure Q versus $\bar{J}$ and τ

2.8 Conclusion

The goal of this task was the development of the elements of a mathematical theory for the representation, analysis and evaluation of organizations. The class of organizations considered were those characterized by distributed information processing and decisionmaking; furthermore, the organizations are supported by command, control, and communications systems.

The integration of n-dimensional information theory with the formalism of Petri Nets and the consideration of cognitive limitations in the decisionmaker's ability to perform his tasks have provided a novel, integrated approach to the problem. A significant contribution of this work is that it has led to the analytical formulation of many organizational design problems that have been considered previously only in an empirical, descriptive context.

3.0 OTHER TASKS

3.1 Decentralized Estimation
(Dr. D. A. Castañon)

Estimation problems with fixed estimator structures, namely distributed estimation problems, were studied by imbedding the estimation in a class of decentralized decisions problems. These decision problems have special structures which can be exploited for some linear Gaussian systems to obtain closed-form solutions for the estimators. In particular, the decision variables do not affect the evolution of the state variables and, in certain cases, they do not affect the observations received by other decisionmakers. Using results from team theory, necessary conditions for optimality of the estimates are derived. For fully decentralized structures, these conditions provide a complete closed-form solution of the estimation problem. These results have been extended to illustrate how the complexity of the local estimation algorithm depends on the importance of correlation between the

errors of the various local estimators.

This task was completed in 1981. A technical paper that presents the significant results of this research effort is included in Appendix B of this report.

3.2 Information Storage and Flow in C^3 Systems
(Ms. E. R. Ducot)

The research objective of this task was twofold: (a) the development of adaptive techniques for modifying and reducing the flow of mission-and engagement-dependent information in a C^3 system operating under stress, and (b) the development of a small expandable software system to support C^3 system research. The task was completed in 1982 with the design and implementation of TECCNET (Testbed for Evaluating Command and Control NETworks). The software has been used for studying the interactions between data and network management strategies and the behavior of the network as seen by the information user.

The design and implementation of TECCNET are described in a technical paper included in Appendix C.

3.3 Distributed Decision Problems in Dymanic Missile Reassignemt Strategies
(Prof. Michael Athans)

In this task, a class of generic air defense problems involving the dynamic stochastic assignment of N defense interceptors against M incoming enemy targets was considered. Key constraints are generated by the assumption that an illuminating radar must reflect energy from each target for a specific amount of time so that the interceptor guidance law can lock on the target.

The modeling phase of this research was completed. Equations were derived that relate one-on-one kill probabilities and other system variables to the overall probability of destroying all incoming targets. Two

stochastic strategies were developed: the shoot-look-launch strategy and the shoot-look-reassign strategy (these include the flexibility of lauching a salvo of interceptors against a target possibly followed by another salvo after kill assessment). Numerical results for optimal stochastic dynamic strategies for simple scenarios were obtained.

The analytical results demonstrated, however, that to solve successfully this class of dynamic optimization problems one needs significant modifications of the available stochastic dynamic programming theory and algorithm.

4. REFERENCES

[1] W. J. McGill, "Multivariable Information Transmission," Psychometrika 19(2) (1954) 97-116.

[2] R. C. Conant, "Laws of Information Which Govern Systems", IEEE Transactions on Systems, Man, and Cybernetics, SMC-6 (1976) 240-255.

[3] C. E. Shannon and W. Weaver, The Mathematical Theory of Communication University of Illinois, Urbana, IL, 1949.

[4] A. I. Khinchin, Mathematical Foundations of Information Theory, Dover, New York, 1957.

[5] T. B. Sheridan and W. R. Ferrel, Man-Machine Systems, MIT Press, Cambridge, MA, 1974.

[6] S. A. Hall and A. H. Levis, "Information Theoretic Models of Memory in Human Decisionmaking Models," in: Proceedings of 6th MIT/ONR Workshop on C^3 Systems, LIDS-P-1300, MIT, 1983 and in Proc. of IXth Congress of the International Federation of Automatic Control, Pergamon Press, Oxford, England, July 1984.

[7] D. A. Stabile and A. H. Levis, "The Design of Information Structures: Basic Allocation Strategies for Organizations, in: Proc. of 6th MIT/ONR Workshop on C^3 Systems, LIDS-P-1313, MIT, 1983; also in Large Scale Systems, No. 6 (1984).

[8] E. E. Lawler, III and J. G. Rhode, "Information and Control in Organizations," Goodyear Publishing Co., Pacific Palisades, Ca, 1976.

[9] D. A. Stabile, "The Design of Information Structures: Basic Allocation Strategies for Organizations," MIT Laboratory for Information and Decision Systems, SM Thesis LIDS-TH-1098, 1981.

[10] K.L. Boettcher and A. H. Levis, "Modeling the Interacting Decisionmaker with Bounded Rationality," IEEE Trans. on Systems, Man, and Cybernetics SMC-12, No. 3, 1982.

[11] K. L. Boettcher and A. H. Levis, "Modeling and Analysis of Teams of Interacting Decisionmakers with Bounded Rationality," Automatica, Vol. 19, No. 6, 1983.

[12] A. H. Levis, and K. L. Boettcher, "Decisionmaking Organizations with Acyclical Information Structures," IEEE Trans. on Systems, Man, and Cybernetics, SMC-13, No. 3, 1983.

[13] S. A. Hall, "Information Theoretic Models of Memory and Storage," MIT, Laboratory for Information and Decision Systems, SM Thesis, LIDS-TH-1232, 1982.

[14] J. G. March, "Bounded Rationality, Ambiguity, and the Engineering of Choice, Bell Journal of Economics, No. 9, 1978.

[15] G. Chyen, "Information Theoretic Models of Preprocessors and Decision Aids," MIT Laboratory for Information and Decision Systems, SM Thesis, LIDS-TH-1377, June 1984.

[17] J. B. Dennis, J. B. Fosseen, and J. P. Linderman, "Data Flow Schemas," in: G. Goos and J. Hartmanis (Eds.), Lecture Notes in Computer Science, Volume 5, International Symposium on Theoretical Programming, Springer-Verlag, Berlin, 1974.

[18] Arvind, V.Kathail, and K. Pingali, "A Dataflow Architecture with Tagged Tokens," MIT Laboratory for Computer Science, Paper No. MIT/LCS-TM-174 1982.

[20] G. Owen, Games Theory, Saunders, Philadelphia, 1968.

APPENDIX A

PUBLICATIONS

A.1 Journal Articles

1. K. L. Boettcher and A. H. Levis, "Modeling the Interacting Decisionmaker with Bounded Rationality," IEEE Trans. on Systems, Man, and Cybernetics, SMC-12, No. 3, May/June 1982.

2. A. H. Levis and K. L. Boettcher, "Decisionmaking Organizations with Acyclical Information Structures," IEEE Trans. on Systems, Man and Cybernetics, SMC-13, No. 3, May/June 1983.

3. K. L. Boettcher and A. H. Levis, "Modeling and Analysis of Teams of Interacting Decisionmakers with Bounded Rationality," Automatica, Vol. 19, No. 6, November 1983.

4. D. A. Castanon, "Decentralized Estimation of Linear Gaussian Systems," to appear in Large Scale Systems, North-Holland Publishing Co.

5. M. Athans, "The Expert Team of Experts Approach to Command and Control (C^2) Organizations, IEEE Control Systems Magazine, Vol. 2, No. 3, Sept. 1982, pp. 30-38.

A.2 Refereed Conference Proceedings

(a) K. L. Boettcher and A. H. Levis, "Modeling the Interacting Decisionmaker with Bounded Rationality," in

- Proc. 4th MIT/ONR Workshop on C^3 Systems, LIDS-R-1159, Vol. IV, MIT, Cambridge, MA, October 1981.
- Control Systems Theory and its Application, Proc. Bilateral Seminar on Control Systems, Science Press, Beijing, PRC, April 1982.

(b) A. H. Levis and K. L. Boettcher, "Decisionmaking Organizations with Acyclical Information Structures," in

- Proc. 5th MIT/ONR Workshop on C^3 Systems, LIDS-R-1267, MIT, Cambridge, MA, December 1982.
- Proc. 21st IEEE Conference on Decision and Control, Orlando, FL, December 1982.

(c) A. H. Levis and K. L. Boettcher, "On the Design of Information Processing and Decisionmaking Organizations," in Optimization of Systems, J. Lions and A. Bensoussan, Eds., Springer Verlag, Berlin, FRG, December 1982.

(d) S. A. Hall and A. H. Levis, "Information Theoretic Models of Memory in Human Decisionmaking Models," in

- Proc. 6th MIT/ONR Workshop on C^3 Systems, LIDS-R-1354, MIT, Cambridge, MA, December 1983.

- Proc. IXth Congress of the International Federation of Automatic Control, Pergamon Press, Oxford, England, July 1984.

(e) D. Tabak and A. H. Levis, "Petri Net Representation of Decision Models," Paper LIDS-P-1386, MIT, Cambridge, MA to appear in

- Proc. 7th MIT/ONR Workshop on C^3 Systems (December 1984).

A.3 Graduate Theses

(a) K. L. Boettcher, "An Information Theoretic Model of the Decision Maker," S.M. Thesis, LIDS-TH-1096, MIT, Cambridge, MA, June 1981.

(b) D. A. Stabile, "The Design of Information Structures: Basic Allocation Strategies for Organizations," S.M. Thesis, LIDS-TH-1098, MIT, Cambridge, MA, June 1981.

(c) S. A. Hall, "Information Theoretic Models of Storage and Memory," S.M. Thesis, LIDS-TH-1232, MIT, Cambridge, MA, August 1982.

(d) G. Chyen, "Information Theoretic Models of Preprocessors and Decision Aids," S.M. Thesis, LIDS-TH-1377, MIT, Cambridge, MA, June 1984.

(e) Y. L. Chow, "Dynamic Missile Reassignment Strategies," S.M. Thesis, Dept. of EECS, MIT (draft available; final draft in preparation).

DECENTRALIZED ESTIMATION OF LINEAR GAUSSIAN SYSTEMS*

by

David A. Castañón†

ABSTRACT

In this paper, we propose a framework for the design of linear decentralized estimation schemes based on a team-theoretic approach. We view local estimates as "decisions" which affect the information received by other decision makers. Using results from team theory, we provide necessary conditions for optimality of the estimates. For fully decentralized structures, these conditions provide a complete closed-form solution of the estimation problem. The complexity of of the resulting estimation algorithms is studied as a function of the performance measure, and in the context of some simple examples.

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1. INTRODUCTION

A standard problem in estimation theory consists of using a set of available information about a random variable to obtain an estimate of its value. When the criterion used in evaluating the estimate is the conditional variance of the estimate, the best estimator is given by the conditional mean. However, this formulation assumes that all of the available information is concentrated at a central location. In many areas of application, such as Command and Control systems and meteorology, the acquisition of data is characterized by sensors which are spatially and temporally distributed. Thus, there are nontrivial costs associated with the transfer of data to a central location for the purpose of estimation.

An approach to designing estimation algorithms for these areas of application is to preprocess some of the data at various local processing nodes, thereby reducing the communication load on the system. The result is an estimation scheme with a fixed structure (often hierarchical), and constraints on the available information at any one node. Figure 1 depicts a typical estimator structure.

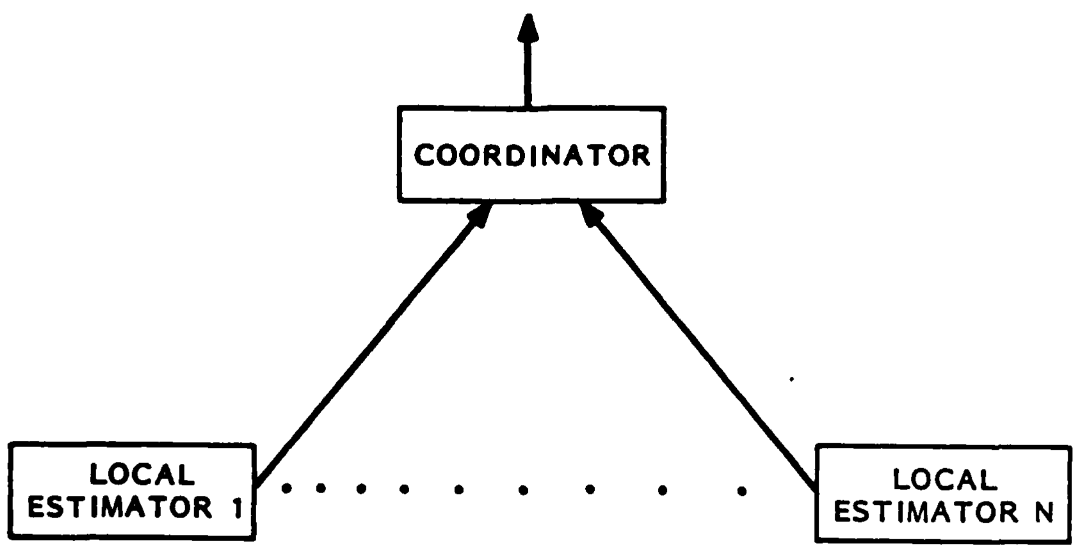


Figure 1

The structure of Figure 1 has similarities with a decentralized decision problem. In this paper, we propose to study estimation problems with fixed estimator structures, hereafter referred to as distributed estimation problems, by imbedding the estimation in a class of decentralized decision problems. These decision problems have special structures which can be exploited for some linear Gaussian systems to obtain closed-form solutions for the estimators. In particular, the decisions variables do not affect the evolution of the state variables and, in certain cases, they do not affect the observations received by other decision makers. This latter case results in a partially nested decision problem, as defined in Ho and Chu [1].

There has been a significant amount of recent work on the subject of distributed estimation. The various approaches can be divided into two classes: The first class consists of methods which use the distributed structure of the problem in such a way as to achieve an overall estimator whose error corresponds to that of a fully centralized estimator, and thus optimality is achieved. Elegant solutions to some of these problems are presented in [2], [3], and [4]. The second class of approaches consists of utilizing a fixed structure, which is simple, to achieve the best performance possible with this restricted structure. This approach can seldom achieve the performance of a centralized scheme. Typical of the results in this case are the papers of Tacker, Sanders and their colleagues [5], [6].

In this paper, we follow the spirit of the second approach. Specifically, we take as given a specific architecture of processing stations, with prespecified flows of information among them. Given this structure, and the apriori statistics of the random variables present in the system, we restrict the data processing to consist of linear strategies of the available data. It is our purpose to characterize the "best" processing schemes in terms of an overall performance measure; our estimation problem will thus become a stochastic team problem, where a number of decision agents with different information seek to minimize a common goal.

Fixed structure decentralized decision problems have been considered by a number of authors [7], [8], and [9]. Our approach in this paper follows very closely the formulation of Barta [9] for linear control of decentralized stochastic systems. Indeed, most of the results of Section 4 of this paper appear in Barta and Sandell [10].

The paper is organized as follows. Section 2 contains the mathematical formulation of fixed structure linear estimation problems using a decision theoretic viewpoint. Section 3 presents general necessary conditions which optimal estimators must statisfy. These conditions are not very useful due to their complexity. In Section 4, we specialize the results of Section 3 to a specific structure which corresponds to a fully decentralized estimation algorithm. This case permits significant analysis, as was previously done in Barta and Sandell []0]. We extend their results to illustrate how the complexity of the local estimation algorithm depends on the importance of correlation between the errors of the various local estimators. Section 5 contains some simple examples which illustrate the results of Section 4. Section 6 discusses the results and areas of future research.

2. MATHEMATICAL FORMULATION

Assume that there are N local substations and one coordinator station in the decentralized estimation systems. Denote the state of the environment by x(t), an $\mathbb{R}^n$-valued random process on [0,T] whose evolution is governed by the stochastic differential equation

$$dx(t) = A(t)x(t)dt + B(t)dw(t), \tag{2.1}$$

where w(t) is an $\mathbb{R}^m$- valued standard Wiener process. Each local substation receives data from local measurements, described by the observation equations

$$dy_i(t) = C_i(t)x(t)dt + D_i(t)dv_i(t) \tag{2.2}$$

where $v_i(t)$, $w(t)$ are standard, mutually independent Wiener processes, and $y_i(t)$ is an R^{m_i} - valued random process. The matrices $A(t)$, $B(t)$, $C_i(t)$, $D_i(t)$ are assumed continuous an $[0,T]$ for $i = 0,\ldots N$. In addition, the matrices $D_i(t)$ are assumed invertible for all i,t.

To each local substation corresponds a decision agent, whose decisions are denoted by $u_i(t)$ in R^{p_i}. The decisions made at each substation depend only on real-time observations of local data, as in equation (2.2), plus the apriori knowledge about the statistics of the systems. The apriori knowledge, common to all local substations and the coordinator station, consists of knowledge of the matrices $A(t)$, $B(t)$, $C_i(t)$, $D_i(t)$, for $i = 0,\ldots N$, $t \in [0,T]$, together with the initial distribution of the initial condition $x(0)$. For the sake of simplicity, we assume that $x(0)$ is a zero-mean, normal random variable with covariance $\Sigma(0)$.

The coordinator station receives the decision outputs of all the local subsystems, $u_i(t)$, $i = 1,\ldots N$, in addition to an independent set of measurements $y_o(t)$. The output of the coordinator station is denoted by $u_o(t)$, and it is based on real-time observation of measurements and the prior decisions of the local substations.

Associated with the estimation structure is a performance index, of the form

$$J = E\left\{\int_o^T (\underline{u}(t) - S(t)x(t))^T Q(t) \ (\underline{u}(t) - S(t)x(t))dt,\right\} \qquad (2.3)$$

where $\underline{u}(t)$ consists of the vector of decisions,

$$\underline{u}^T(t) = (u_o^T(t),\ldots,u_N^T(t)), \qquad (2.4)$$

and the superscript T denotes transposition. The matrix $Q(t)$ is assumed positive semidefinite and continuous for t in $[0,T]$. With this performance criterion, the design of a distributed estimation scheme can be reduced to determining the admissible decision strategies which minimize the quadratic function J.

The admissible strategies are restricted to be linear maps of the available information which yield mean-square integrable decision variables. Specifically, since equation (2.2) implies that the local observations are corrupted by additive white noise, we assume that, for i = 1,...n,

$$u_i(t) = \int_o^t H_i(t,s)dy_i(s) \tag{2.5}$$

where

$$H_i(t,s) = 0 \quad \text{if } s > t, \tag{2.6}$$

and

$$\text{Trace} \int_o^T \int_o^T H_i(t,s)H_i^T(t,s)dtds < \infty. \tag{2.7}$$

For the coordinator, we assume that

$$\begin{aligned} u_o(t) = \int_o^T H_o(t,s)dy_o(s) + \sum_{i=1}^{N}\int_o^t K_i(t,s)u_i(s)ds \\ + \sum_{i=1}^{n} L_i(t)\ u_i(t) \end{aligned} \tag{2.8}$$

where H_o, K_i satisfy (2.6) and (2.7), while the matrices $L_i(t)$ are continuous on [0,T].

The parametrization of the control laws in equations (2.5) to (2.8) results in admissible strategy spaces which are Hilbert spaces. Specifically, the admissible strategies for u_i, i = 1,...N, are elements of the Hilbert space of linear operators from $L_2([0,T], R^{n_i})$ to $L_2([0,T], R^{p_i})$ with finite trace, and inner product

$$\langle H^1, H^2\rangle = \text{Trace} \int_o^T \int_o^T H^1(t,s)H^{2^T}(t,s)dtds = \text{Trace}\ (H_1H_2^*). \tag{2.9}$$

For additional information about Hilbert spaces of operators, the reader should consult Balakrishnan [11]. We will use the symbol H_i without its arguments to refer to the linear operator, while $H_i(t,s)$ will be used to refer to the kernel of the operator.

The assumption of linear strategies for all decision agents in the problem represents a restriction on the class of admissible strategies. However, the system and observations described by equations (2.1) and (2.2) result in zero-mean, jointly Gaussian random processes $x, y_o, \ldots y_N$. Since the decisions $\underline{u}(t)$ do not affect the evolution of the state $x(t)$ (this is a property of estimation problems) for any control law $\underline{u}(t)$ such that

$$E\int_0^T ||\underline{u}(t)||^2 dt < \infty, \tag{2.10}$$

we can use a version of Fubini's theorem to show

$$J = \int_0^T E\left\{(\underline{u}(t)-S(t)x(t))^T Q(t)(\underline{u}(t)-S(t)x(t))\right\}dt. \tag{2.11}$$

Notice that the optimal estimator will minimize the integrand

$$J_1 = E\left\{(\underline{u}(t)-S(t)x(t))^T Q(t)(\underline{u}(t)-S(t)x(t))\right\} \tag{2.12}$$

almost everywhere. In many cases, this will enable us to show that the true optimal solution belongs to the admissible class of linear strategies. To conclude this section, we will discuss some relevant examples, and indicate how they fit in this framework.

<u>Example 1</u>: Centralized estimation

Assume that $N = 0$, so that the only station present is the coordinator station. In this case, J_1 corresponds to

$$J_1 = E\left\{(u_o(t)-S(t)x(t))^T Q(t)(u_o(t)-S(t)x(t))\right\}.$$

Its minimum among all mean-square integrable $u_o(t)$ is achieved at

$$u_o(t) = S(t)\hat{x}(t) \tag{2.13}$$

where $\hat{x}(t)$ is the minimum variance estimate of x_t, given the prior observations, which is obtained from a Kalman filter. Hence, the optimal estimator is linear.

Example 2. Hierarchical Estimation

Let $N = 2$. Furthermore, let $p_0 = p_1 = p_2 = n$ and

$$S(t) = \begin{bmatrix} I \\ I \\ I \end{bmatrix} \qquad Q(t) = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}.$$

Assume $C_o(t) \equiv 0$.

Then, equation (2.12) yields

$$J_1 = E\left\{\sum_{i=0}^{2} (u_i(t)-x(t))^T(u_i(t)-x(t))\right\}.$$

We consider the minimization of J_1 over all mean-square integrable decision. The last two terms in the sum are minimized by using local Kalman filters at each local substation. Furthermore, it was established in W:.llsky, Castanon et al [2], that the first term can be minimized absolutely, when the local strategies are Kalman filters, by a strategy of the form (2.8). Hence, the optimal hierarchical estimator for this problem is in the class of linear estimators.

Example 3. Fully Decentralized Estimation

Assume that there is no coordinator station, so that $u_o(t) \equiv 0$ for all t. In this case,

$$J_1 = E\left\{(\underline{u}(t)-s(t)x(t))^T\ Q(t)\,(\underline{u}(t)-s(t)x(t))\right\}.$$

For each t, this is a static team problem with jointly Gaussian statistics; hence, Radner's theorem [12] implies that the optimal decision strategies are linear maps of the available observations, and hence they belong to the linear class in equations (2.5) to (2.8).

Example 4. Let $N = 1$, $p_1 = 1$, $p_o = n$, and

$$S(t) = \begin{bmatrix} I \\ 0 \end{bmatrix}, \qquad Q(t) = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}.$$

Then,

$$J_1 = E\left\{(u_o(t)-x(t))^T(u_o(t)-x(t))\right\}$$

It is clear that, if n > 1, some form of nonlinear encoding of the information y_1 will provide a lower value of J_1 than the best linear encoder, because u_1 is a scalar signal and x is a vector process. In this case, the optimal decision rules are nonlinear.

In many cases, the optimal estimation strategies will be nonlinear. Nevertheless, there will be a person-by-person-optimal linear strategy which will be of interest because of ease of implementation. In the next Section, we provide necessary conditions which characterize these linear person-by-person optimal strategies.

3. NECESSARY CONDITIONS

The formulation of Section 2 imbedded the distributed estimation problem into a team decision problem with a quadratic criterion, where decision rules are elements of a Hilbert space of linear operators. In this section, we provide necessary conditions which characterize the estimators resulting from this approach. The mathematical development of this section follows closely the development in Barta [9].

In operator notation, equations (2.5) and (2.8) can be written as

$$u_i = H_i(dy_i), \qquad i = 1,\ldots N \tag{3.1}$$

$$u_o = H_o dy_o + \sum_{i=1}^{N}(K_i u_i + L_i u_i) \tag{3.2}$$

where L_i is the linear operator with kernel

$$L_i(t,s) = L_i(t)\delta(t-s) \tag{3.3}$$

Furthermore, the quadratic functional (2.4) can be written as

$$J = E\left\{\int_0^T (\underline{u}(t)-S(t)x(t))^T Q(t)(\underline{u}(t)-S(t)x(t))dt\right\}$$

$$= \text{Trace}\left\{S^*QS\textstyle\sum_{xx} + Q\sum_{\underline{\underline{u}}\underline{\underline{u}}} - 2Q\sum_{\underline{u}x} S^*\right\} \tag{3.4}$$

where $\sum_{xx}$, $\sum_{\underline{u}x}$, and $\sum_{\underline{\underline{u}}\underline{\underline{u}}}$ are the covariance operators [11] corresponding to the random processes $x(t)$ and $\underline{u}(t)$. Note that the decision operators are implicit in defining $\underline{u}(t)$ as a random process.

Let's partition $\underline{u}$ as

$$\underline{u}(t) = [u_o(t) \mid u_1(t) \ldots u_N(t)]^T = [u_o(t), \bar{u}(t)]^T \tag{3.5}$$

Then, $\sum_{\underline{\underline{u}}\underline{\underline{u}}}$ can be partitioned

$$\sum_{\underline{\underline{u}}\underline{\underline{u}}} = \begin{bmatrix} \sum_{u_o u_o} & \sum_{u_o \bar{u}} \\ \sum^*_{u_o \bar{u}} & \sum_{\bar{u}\bar{u}} \end{bmatrix} \tag{3.6}$$

Furthermore, $\bar{u}(t)$ is related to $\underline{y}(t)$ by

$$\bar{u}(t) = \begin{bmatrix} H_1 & & \\ & \ddots & \\ & & H_N \end{bmatrix} \underline{y}(t) \tag{3.7}$$

$$= \text{diag}\{H_i\} \; \underline{y}(t)$$

so that

$$\sum_{\bar{u}\bar{u}} = (\text{diag } H_i) \textstyle\sum_{d\underline{y}d\underline{y}} (\text{diag } H_i)^* \tag{3.8}$$

Similarly,

$$\sum_{u_o\bar{u}} = \left[H_o \textstyle\sum_{dy_o dy_1} H_1^* \ldots H_o \sum_{dy_o dy_N} H_N^*\right] +$$

$$+\sum_{i=1}^{N} [(K_i+L_i)H_i \sum_{dy_i dy_1} H_1^* \cdots (K_i+L_i)H_i \sum_{dy_i dy_N} H_N^*] \tag{3.9}$$

and

$$\sum_{u_o u_o} = H_o \sum_{dy_o dy_o} H_o^* + \sum_{i=1}^{N} \{H_o \sum_{dy_o dy_i} H_i^*(K_i^* + L_i^*)$$

$$+(K_i+L_i)H_i \sum_{dy_o dy_i}^{*} H_o^*\}$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{N} (K_i+L_i)H_i \sum_{dy_i dy_j} H_j^* (K_j+L_j)^* \tag{3.10}$$

A similar partition yields

$$\sum_{\underline{u}x} = \begin{bmatrix} \sum_{u_o x} \\ \\ \sum_{\bar{u}x} \end{bmatrix} \tag{3.11}$$

where

$$\sum_{u_o x} = H_o \sum_{dy_o x} + \sum_{i=1}^{N} (K_i+L_i)\ H_i \sum_{dy_i x} \tag{3.12}$$

$$\sum_{\bar{u}x} = [H_i \sum_{dy_1 x} \cdots H_N \sum_{dy_N x}] \tag{3.12}$$

Using equations (3.6) - (3.13) in equation (3.4), we can express the functional J as a deterministic quadratic function of the operators H_i, L_i, K_i, which are elements of a linear Hilbert space. We will denote this dependence by

$$J = J\ (\bar{H},\ \bar{L},\ \bar{K}) \tag{3.14}$$

Since J is a quadratic functional, and the linear operators $\bar{H}$, $\bar{L}$, $\bar{K}$ are elements of Hilbert spaces, we can compute the Frechet differential of J [13] with respect to variations in the operators. In particular, we will denote the Frechet differential of J in the direction of each of the components of $\underline{H}$, $\underline{K}$ and $\underline{L}$. Partition the operators Q, S, according to equations (3.6), as

$$Q = \begin{pmatrix} Q_{oo} & Q_{o1} \\ Q_{1o} & Q_{11} \end{pmatrix} \tag{3.14}$$

$$S = \begin{pmatrix} S_o \\ S_1 \end{pmatrix} \tag{3.15}$$

Then, we can use equations (3.6) - (3.15) to obtain the Frechet differentials:

$$\begin{aligned} \delta_{H_o} J(\bar{H},\bar{K},\bar{L},\tilde{H}_o) = 2 \text{ Trace } \Bigg\{ \Bigg[& Q_{oo} H_o \textstyle\sum_{dy_o dy_o} + \sum_{i=1}^{N} Q_{oo}(K_i+L_i) H_i \sum^*_{dy_o dy_i} \\ & + Q_{o1} \begin{bmatrix} H_1 \sum^*_{dy_o dy_1} \\ H_N \sum^*_{dy_o dy_N} \end{bmatrix} - Q_{oo} S_o \textstyle\sum^*_{dy_o x} \\ & - Q_{o1} S_1 \textstyle\sum^*_{dy_o x} \Bigg] \tilde{H}^*_o \Bigg\} \end{aligned} \tag{3.16}$$

$$\begin{aligned} \delta_{K_i+L_i} J(\bar{H},\bar{K},\bar{L};\tilde{K}_i+\tilde{L}_i) = 2 \text{ Trace } \Bigg\{ \Bigg[& Q_{oo} H_o \textstyle\sum_{dy_o dy_i} H^*_i \\ & + Q_{oo} \sum_{j=1}^{N} (K_j+L_j) H_j \textstyle\sum_{dy_j dy_i} H^*_i + Q_{o1} \begin{bmatrix} H_1 \sum^*_{dy_i dy_1} H^*_i \\ \vdots \\ H_N \sum^*_{dy_i dy_N} H^*_i \end{bmatrix} \\ & - Q_{oo} S_o \textstyle\sum^*_{dy_i x} H^*_i - Q_{o1} S_1 \sum^*_{dy_i x} H^*_i \Bigg] (\tilde{K}^*_i + \tilde{L}^*_i) \Bigg\} \end{aligned} \tag{3.17}$$

$$\delta_{H_i} J(\bar{H},\bar{K},\bar{L} \; ; \; \tilde{H}_i) = 2 \text{ Trace} \Bigg\{ \Bigg[(K_i^* + L_i^*) Q_{oo} H_o \textstyle\sum_{dy_o dy_i}$$

$$+ \sum_{j=1}^{N} (K_i + L_i)^* \; Q_{oo} (K_j + L_j) \; H_j \textstyle\sum_{dy_j dy_i}$$

$$+ Q_{1o}^{i} \; H_o \textstyle\sum_{dy_o dy_i} + (K_i + L_i)^* \begin{bmatrix} Q_{o1}^{i} \; H_1 \; \sum^*_{dy_i dy_1} \\ \\ Q_{o1}^{N} \; H_N \; \sum^*_{dy_i dy_N} \end{bmatrix}$$

$$+ Q_{1o}^{i} \sum_{j=1}^{N} (K_j + L_j) H_j \textstyle\sum_{dy_j dy_i} + \displaystyle\sum_{j=1}^{N} Q_{11}^{ij} \; H_j \textstyle\sum_{dy_j dy_i}$$

$$- (K_i + L_i)^* (Q_{oo} S_o + Q_{o1} S_1) \textstyle\sum^*_{dy_i x}$$

$$-(Q_{1o} S_o + Q_{11} S_1)^{i} \textstyle\sum^*_{dy_i x} \Bigg] \tilde{H}_i^* \Bigg\} \tag{3.18}$$

where Q_{1o}^{i}, Q_{11}^{ii}, S^{i} are the blocks partition in the corresponding partition of $\bar{u}(t) = (u,(t),\ldots u_N(t))^T$.

Using expressions (3.16) - (3.18), we can provide necessary conditions for optimality of a set of linear maps $(\bar{H},\bar{K},\bar{L})$, as follows:

<u>Proposition 3.1</u> If $\bar{H},\bar{K},\bar{L}$ minimize the functional J over the space of all linear maps, then

(a) $\delta J_{H_o}(\bar{H},\bar{K},\bar{L} \; ; \; \tilde{H}_o) = 0$

$\delta J_{K_i+L_i}(\bar{H},\bar{K},\bar{L} \; ; \; \tilde{K}_i + \tilde{L}_i) = 0$

$\delta J_{H_i}(\bar{H},\bar{K},\bar{L} \; ; \; \tilde{H}_i) = 0$

for all i=1,...N, and for all admissible $\tilde{K}_i, \tilde{L}_i, \tilde{H}_i$ and $\tilde{H}_o$.

Proof The proof follows directly from Theorem 1 in Chapter 7 in Luenberger [13], since the existence of Frechet differentials provides an expression for the Gateaux differential, which must be zero at a minimum.

Proposition 3.1 can be used, together with the fact that admissible operators $\bar{H}, \bar{K}, \bar{L}$ are Volterra integral operators, to obtain sets of coupled integral conditions which characterize the optimal solution, in a manner similar to Wiener-Hopf factorization [14]. We will not do so here, focusing instead on obtaining the expressions which characterize the optimum in the specific case of equations (2.2) - (2.3) for the fully decentralized case in the next section.

4. FULLY DECENTRALIZED ESTIMATION

In the fully decentralized case, the coordinator station is absent. In terms of the formulation of section 3, the operators K_i, L_i and H_o are identically zero, as are the weighting matrices S_o, Q_{oo}, Q_{o1} and Q_{1o}, for all time t in [0,T]. This causes an extensive simplification in the equations of Proposition 3.1. Specifically, equation (3.18) now becomes

$$\delta_{\tilde{H}_i} J(\bar{H};\tilde{H}_i) = \text{Trace}\left\{\left[\sum_{j=1}^{N} 2Q_{11}^{ij} H_j \Sigma_{dy_j dy_i} - (Q_{11}S_1)^i \Sigma^*_{dy_i x}\right] \tilde{H}_i^*\right\} \quad (4.1)$$

The equivalent set of integral equations corresponding to equation (4.1) are

$$\sum_{j=1}^{N} Q_{11}^{ij} \int_0^t H_j(t,s_1) \Sigma_{dy_j dy_i}(s_1,s) ds_1 = (Q_{11}S_1(t))^i \Sigma_{xdy_i}(t,s) \quad (4.2)$$

A similar equation can be found in Barta-Sandell [10], where a solution is found using an innovations approach. We will present a different derivation of their results in this section.

Assume that $Q_{11} > 0$ and is constant in time. This implies that the cost functional J is strictly convex, so that there is a unique minimum, which is characterized by the integral equation (4.2). Furthermore, assume, without loss of generality, that all decisions u_i are scalar-valued, that is $p_i = 1$ for all i. A vector-valued decision can be decomposed into p_i stations with the same information. Hence, the assumption in equation (2.3) that the v_i are mutually independent Wiener processes will be removed at this stage, to allow for this development.

We begin by noting that equation (4.2) is a linear equation driven by a sum of terms in the right hand side. Hence, by superposition, the optimal solution $\hat{H}_j(t,s)$ can be written as

$$\hat{H}_j(t,s) = \sum_{\ell=1}^{N}\sum_{k=1}^{n} G_j^{\ell k}(t,s)\; S_{\ell k}(t) \tag{4.3}$$

where $G_j^{\ell k}(t,s)$ minimizes J when $S = \delta_{\ell k}$, that is, it has a one in the ℓk th entry and zero elsewhere. Hence, $G_j^{\ell k}(t,s)$ solves

$$\sum_{j=1}^{N} Q_{11}^{ij} \int_0^t G_j^{\ell k}(t,s_1)\Sigma_{dy_j dy_i}(s_1,s)ds_1 = Q_{11}^{i\ell}\,\Sigma_{x_k dy_i}(t,s) \tag{4.4}$$

Notice that the form of Q determines the form of the linear system on the left side. It is possible to solve for all $G_j^{\ell k}$ simultaneously, because of the consistency of the problems (4.4). Let $J^{\ell k}$ denote the cost function J when $s = \delta_{\ell k}$. Then,

$$(G_1^{\ell k}, \ldots G_n^{\ell k}) = \underset{\bar{G}^{\ell k}}{\operatorname{argmin}}\; J^{\ell k}(\bar{G}^{\ell k}) \tag{4.5}$$

Define a global cost J^T, given by

$$J^T(\bar{G}^{1,1},\ldots\bar{G}^{N,n}) = \sum_{\ell=1}^{N}\sum_{k=1}^{N} J^{\ell k}(\bar{G}^{\ell k}) \tag{4.6}$$

The cost J^T is separable in its arguments. Hence, minimization of J^T corresponds to solving equation (4.5) for each ℓ,k.

Let's examine closely the nature of the costs $J^{\ell k}$. From equation (2.4), $J^{\ell k}$ corresponds to

$$J^{\ell k} = E\left\{\int_0^T (\underline{u}(t) - \delta_{i-\ell} x_k(t))^T Q\, (\underline{u}(t) - \delta_{i-\ell} x_k(t))dt\right\} \tag{4.7}$$

where $\delta_{i-\ell}$ is a vector with all zeroes except a one in the ℓ'th entry. Furthermore, minimization of $J_\ell^{\ell k}$ is accomplished by minimizing

$$J_1^{\ell k} = E \quad (\underline{u}(t) - \delta_{i-\ell} x_k(t))^T Q(\underline{u}(t) - \delta_{i-\ell} x_k(t)) \tag{4.8}$$

for each t. Let $d_i(t)$ correspond to the n x N matrix

$$d_i(t) = \begin{pmatrix} \underline{u}^{i1} \\ \vdots \\ \underline{u}^{in} \end{pmatrix} \tag{4.9}$$

representing the decision variables associated with problems J_1^{ik}, k=1,...n in (4.6). Let D(t) be

$$D(t) \equiv \begin{bmatrix} d_1(t) \\ \vdots \\ d_N(t) \end{bmatrix}$$

Let

$$X(t) = \begin{bmatrix} x(t) & & & \\ & x(t) & & \\ & & \ddots & \\ & & & x(t) \end{bmatrix}$$

be an n N x N matrix. Then, a simple calculation establishes that

$$J_1^T = \text{Trace } E\left\{(D(t)-X(t))\ Q(D(t)-X(t))^T\right\} \tag{4.10}$$

where the i-th column of D(t) is a linear function of the local observation process $y_i(t)$ only.

This is the same formulation used in Barta-Sandell [10]. We will state their main result without proof, as it applies to systems of the form (2.2) - (2.4). Before we can do so, we must introduce some notation.

The state process of equation (2.2) is given by

$$dx(t) = A(t)\, x(t)dt + B(t)dw(t) \tag{4.11}$$

with local observations

$$dy_i(t) = C_i(t)\, x(t)dt + D_i(t)dv_i(t) \tag{4.12}$$

where $v_i(t)$, $w(t)$ are standard Brownian motions with $w(t)$ independent of all $v_i(s)$.

Let

$$\mathcal{A}(t) = \mathrm{diag}\left\{A(t),\ldots A(t)\right\}$$

$$\mathcal{B}(t)dw(t) = \mathrm{diag}\left\{B(t)dw(t),\ldots B(t)dw(t)\right\}$$

$$C(t) = \mathrm{diag}\left\{C_1(t),\ldots C_n(t)\right\} \tag{4.13}$$

then, we have

$$dX(t) = \mathcal{A}(t)X(t)dt + \mathcal{B}(t)dw(t) \tag{4.14}$$

Define also

$$\sum_{ww}(t) = \begin{bmatrix} Q_{11}I & \cdots & Q_{1N}I \\ \vdots & & \\ Q_{N1}I & \cdots & Q_{NN}I \end{bmatrix} \cdot \mathrm{diag}\left[B(t)B^T(t),\ldots B(t)B^T(t)\right] \tag{4.15}$$

as the enlarged system relevant driving noise intensity.

Similarly, define

$$\Sigma_{vv}(t) = \begin{bmatrix} Q_{11}D_1(t)D_1^T(t) & \cdots & Q_{1N}E\left\{D_1(t)dv_1(t)dv_N(t)D_N^T(t)\right\} \\ \\ Q_{N1}\ E\left\{D_N(t)dv_N(t)\ dv_1(t)D_N^T(t)\right\} & \cdots & Q_{NN}D_N(t)D_N^T(t) \end{bmatrix} \quad (4.16)$$

as the enlarged system relevant observation noise intensity. With this notation, the main result of [10] is:

Proposition 4.2 The Decentralized Kalman Filter

The optimal team decision rule for equation (4.10), $\hat{X}(t)$, satisfies

$$d\hat{X}_i(t) = A(t)\ \hat{X}_i(t)dt + K(t)\ [I_i dy_i(t) - C(t)\hat{X}_i(t)] \quad (4.17)$$

where

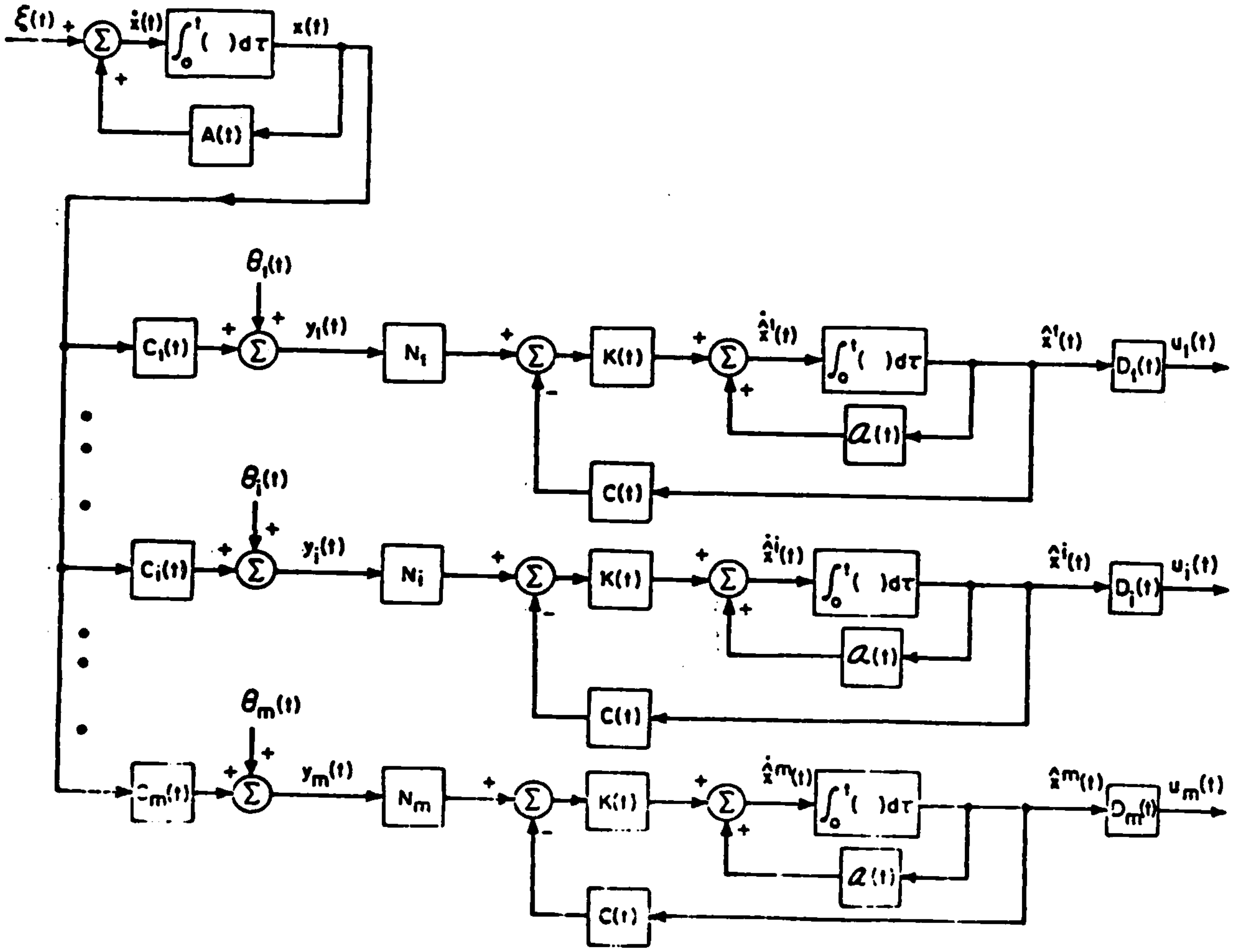
$$K(t) = \Sigma(t)\ C'(t)\ \Sigma_{vv}^{-1}(t)$$

$I_i = [0^T, \ldots I^T, \ldots 0^T]^T$ is a $\sum_{j=1}^{M} m_j \times m_i$ dimensioned matrix with the identity in its ith block, and $\Sigma(t)$ solves the Ricatti equation

$$\dot{\Sigma} = A(t)\Sigma + \Sigma A^T(t) - K(t)\Sigma_{vv}K^T(t) + \Sigma_{ww} \quad (4.16)$$

$$\Sigma(o) = \begin{bmatrix} Q_{11}I & \cdots & Q_{1N}I \\ \vdots & & \\ Q_{N1}I & \cdots & Q_{NN}I \end{bmatrix} \quad \text{diag}\ [\Sigma_o, \ldots \Sigma_o].$$

The estimator of Proposition 4.2 is depicted in Figure 2. The striking feature of this estimator is that each local agent uses identical estimation systems, of dimension NnxN, differing only in the input used to drive the systems. However, in many applications, these estimators are much larger than are necessary. In particular, it is important to note that it is the presence of Q which creates nontrivial couplings in the team problem, leading to large-dimension estimators.



ξ(t)
ẋ(t)
x(t)
A(t)
θ1(t)
y1(t)
C1(t)
N1
K(t)
a(t)
C(t)
D1(t)
u1(t)
θi(t)
yi(t)
Ci(t)
Ni
Di(t)
ui(t)
θm(t)
ym(t)
Cm(t)
Nm
Dm(t)
um(t)

Figure 2

When Q is diagonal, the expresions for $\sum_{ww}(t)$ and $\sum_{vv}(t)$ are block-diagonal. In this case, it can be established that $\sum(t)$, as given by equation (4.16) will also be block-diagonal, and the optimal estimator will decompose into blocks of much smaller dimension. We formalize this in the following proposition.

Proposition 4.3 Assume Q is diagonal. Then, the optimal decision rule which minimizes (4.10) can be synthesized using n-dimensional estimators at each local station.

The proof follows directly from equations (4.15) and (4.16). In the next section, we will study some specific examples to illustrate the comlexity of the algorithm of Proposition 4.2, and the relation of the off-diagonal elements of the matrix Q with this complexity.

5. EXAMPLES

In this section, we discuss some examples of fully decentralized estimation problems, indicating their relation with the results of section 4. To facilitate the understanding of the examples, we will discuss only non-dynamic Gaussian systems.

Example 1. Let x_1, x_2 be independent, zero-mean Gaussian random variables with unit variance. Define the two observation equations

$$y_1 = x_1 + v_1 \tag{5.1}$$

$$y_2 = x_2 + v_2 \tag{5.2}$$

where v_1, v_2, x_1, x_2 are mutually independent, normal, zero-mean random variables with unit variance.

Assume that there are two local substations. Each substation i has access to its own measurement y_i. The performance of the elements is to be evaluated as

$$J = E\left\{\left[\begin{pmatrix}u_1\\u_2\end{pmatrix} - \begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}\right]^T \begin{pmatrix}1 & 1/2\\1/2 & 1\end{pmatrix}\left[\begin{pmatrix}u_1\\u_2\end{pmatrix} - \begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}\right]\right\}$$

$$= E\left\{(u_1-x_1-x_2)^2 + (u_2-x_2)^2 + (u_1-x_1-x_2)(u_2-x_2)\right\}. \tag{5.3}$$

Conditioning on y_1 inside the expectation of equation (5.3), and differentiating with respect to u_1 yields

$$2u_1 - 2\,E\,\{x_1|y_1\} = 0$$

Similarly, conditioning on y_2 and differentiating with respect to u_2 yields

$$2u_2 - 2E\,\{x_2|y_2\} - E\,\{x_2|y_2\} = 0$$

Hence,

$$u_1 = E\,\{x_1|y_1\} \tag{5.4}$$

$$u_2 = \frac{3}{2}\,E\,\{x_2|y_2\}$$

In this example, $S = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$. If $S = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, it is clear that

$$u_1 = E\,\{x_1|y_1\} \tag{5.5}$$

$$u_2 = E\,\{x_2|y_2\}$$

is the optimal decentralized estimator. Now, let $S = \begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}$.

Then,

$$J = E\,\{(u_1-x_2)^2 + u_2^2 + (u_1-x_2)u_2\}$$

conditioning with respect to y_1 and differentiating with respect to u_1 yields

$$2u_1 = 0$$

Repeating for y_2 and u_2 yields

$$2u_2 - E\{x_2|y_2\} = 0$$

Hence, for $S = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, the optimal strategy is

$$u_1 = 0$$

$$u_2 = 1/2 \;\; E\{x_2|y_2\}.$$

As indicated in Section 4, the solution for $S = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ is the superposition of the solutions for $S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $S = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$.

The presence of the off diagonal elements of $Q = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}$ is important in creating the nature of the solution. Notice that, in spite of the independence x_1, y_1 and x_2, y_2, that the optimal estimator for $S = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is <u>not</u>

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{bmatrix} E\{x_1|y_1\} \\ E\{x_2|y_2\} \end{bmatrix}$$

<u>Example 2</u>

Assume, in example 1, that $x_1 = x_2$. Repeating the same logic for obtaining the optimal solution for $S = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, we obtain the sufficient conditions

$$\begin{aligned} 2u_1 - 5 \;\; E\{x_1|y_1\} + E\{u_2|y_1\} &= 0 \\ 2u_2 - 4 \;\; E\{x_2|y_2\} + E\{u_1|y_2\} &= 0 \end{aligned} \qquad (5.5)$$

the coupled equations (5.5) can be solved by noting that $u_1 = ay_1$, $u_2 = by_2$, for some constant a, b. Equation (5.5) becomes

$$2u_1 - 5\, E\{x_1|y_1\} + bE\{y_2|y_1\} = 0$$

$$2u_2 - 4\, E\{x_2|y_2\} + aE\{y_1|y_2\} = 0 \tag{5.6}$$

Now,

$$E\{y_2|y_1\} = E\{x_1|y_1\}$$

$$E\{y_1|y_2\} = E\{x_2|y_2\}$$

So,

$$a\ y_1 = \left(\frac{5}{2} - \frac{b}{2}\right) E\{x_1|y_1\}$$

$$b\ y_2 = 2 - a/2\ E\{x_2|y_2\} \tag{5.7}$$

Rewriting in terms of contants,

$$a + b/4 = 5/4$$

$$b + a/4 = 1$$

so

$$a = 16/15 \tag{5.8}$$
$$b = 11/15$$

Equation (5.8) was obtained by solving the simultaneous euqations obtained from the variational arguments. For differential systems, these equations will be coupled integral equations which are hard to solve.

Let's establish the solution (5.8) using the decomposition approach of Section 4. Let $S = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. Then, the performance measure is

$$J = E\{(u_1-x_1)^2 + (u_1-x_1)u_2+u_2^2\}$$

Variational arguments yield

$$2u_1 - 2\,E\{x_1|y_1\} + E\{u_2|y_1\} = 0$$

$$2u_2 - E\{x_2|y_2\} + E\{u_1|y_2\} = 0 \qquad (5.9)$$

which imply $a = 7/15$, $b = 2/15$

By symmetry, the solution for $S = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ is

$$a = 2/15 \qquad b = 7/15$$

For $S = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, the performance measure is

$$J = E\{(u_1 - x_2)^2 + (u_1 - x_2)u_2 + u_2^2\}$$

$$= E\{(u_1 - x_1)^2 + (u_1 - x_1)u_2 + u_2^2\}$$

which has already been solved, yielding

$$a = 7/15 \qquad b = 2/15$$

Summary all three yields the result for $S = \begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix}$ as

$$a = 7/15 + 2/15 + 7/15 = 16/15$$

$$b = 4/15 + 7/15 = 11/15$$

We will now use proposition 4.2 directly to solve example 2. Since $x_1 = x_2$, the effective state dimension is 1. Hence, the matrix D in Section 4 has dimension 2 x 2, with the first column a function of y_1, while the second column is a fucntion of y_2. The overall team cost is given as in (4.10), by

$$J = \text{Trace}\left[\left(D - \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix}\right) \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix} \left(D - \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix}\right)^T \right]$$

The optimal solution $\hat{X}$ is characterized by

$$E\left\{(\hat{X} - \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} Q\, D^T\right\} = 0 \tag{5.10}$$

for any D whose first column is a function of y_1, and its second column a function of y_2. Let

$$\hat{X} = \begin{pmatrix} a_1y_1 & b_1y_2 \\ a_2y_1 & b_2y_2 \end{pmatrix} \tag{5.11}$$

Equations (5.10) and (5.11) imply

$$E\left\{\begin{pmatrix} a_1y_1-x & b_1y_2 \\ a_2y_1 & a_2y_2-x \end{pmatrix} Q \begin{pmatrix} y_1 & 0 \\ 0 & y_2 \end{pmatrix}\right\} = 0 \tag{5.12}$$

which reduces to

$$\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} E\left\{\begin{pmatrix} y_1 & 0 \\ 0 & y_2 \end{pmatrix} Q \begin{pmatrix} y_1 & 0 \\ 0 & y_2 \end{pmatrix}\right\} = E\left\{\begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} Q \begin{pmatrix} y_1 & 0 \\ 0 & y_2 \end{pmatrix}\right\} \tag{5.13}$$

Let's compute the terms in equations (5.13).

$$E\left\{\begin{pmatrix} y_1 & 0 \\ 0 & y_2 \end{pmatrix}\begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}\begin{pmatrix} y_1 & 0 \\ 0 & y_2 \end{pmatrix}\right\} = \begin{pmatrix} 2 & 1/2 \\ 1/2 & 2 \end{pmatrix}$$

$$E\left\{\begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} Q \begin{pmatrix} y_1 & 0 \\ 0 & y_2 \end{pmatrix}\right. = \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}$$

Hence,

$$\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} = \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1/2 \\ 1/2 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 7/15 & 2/15 \\ 2/15 & 7/15 \end{pmatrix}$$

The solution for $S = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is thus

$$u_1 = (2 \cdot 7/15 + 2/15)\, y_1 = 16/15\, y_1$$

$$u_2 = (2 \cdot 2/15 + 7/15)\, y_2 = 11/15\, y_2,$$

as was established before.

Notice that a diagonal Q would have decoupled the problem by permitting a trivial inversion of a diagonal matrix, as predicted in proposition 4.3.

6. CONCLUSION

We have presented a framework for the design of distributed estimation schemes with specific architectures, based on a decision theoretic approach. For a fully decentralized architecture, explicit solutions to the estimation problem were described and illustrated with several examples. The examples illustrate that the complexity of the decentralized estimation scheme is critically dependent on the importance of the cross-correlation of errors in the local estimators, which are represented by the off-diagonal elements of the positive definite matrix Q. Most practical systems will want to weigh heavily the correlation of local errors. For example, in a distributed surveillance network, it is important that errors in location or detection at one local substation be corrected by other substations. In other words, it is very costly for all substations to err in the same way. This is reflected in the performance measure by the off-diagonal elements of Q.

The examples in Section 5 illustrate the high dimensionality required by the local estimators in order to compensate for correlations in their errors. It is our conjecture that the dimensionality of the local estimators is directly related to the number of off-diagonal elemets of Q.

When there is a coordinator station present, the results presented in Section 3 provide necessary conditions for the optimality of the estimation operators. Unfortunately, the coupling between decisions at the local sub-stations and the information available to the coordinator makes the analysis a difficult problem. We expect that, under some simplifying assumptions, the necessary conditions of Section 3 can lead to a solution, as in Section 4. Such results have been reported in Willsky, Castanon et al [2] for a simple class of performance measures.

The formulation of Section 2 can be extended to incorporate communication restrictions, as well as delays in the transmission of local decisions. These are areas which will be studied in the future.

REFERENCES

[1] Y. C. Ho, K. C. Chu, " Equivalence of Information Structure in Static and Dynamic Team Problems," IEEE Trans. on Auto. Control, Vol., AC-18, 1973.

[2] A. S. Willsky, M. Bello, D. Castanon, B. C. Levy, G. Verghese, "Combining and Updating of Local Estimates and Regional Maps Along Sets of One-Dimensional Tracks," to appear in IEEE Trans. on Auto. Control.

[3] J. L. Speyer, "Computation and Transmission Requirements for a Decentralized L Q G Control Problem," IEEE Trans. Auto.Control, Vol. AC-24, No. 2, April 1979.

[4] B. C. Levy, D. A. Castanon, G. C. Verghese, A. S. Willsky, " A Scaterring Framework for Decentralized Estimation Problems," MIT/LIDS paper 1075, March 1981, Submitted to Automatica.

[5] E. C. Tacker and C. W. Sanders, "Decentralized Structures for State Estimation in Large Scale Systems," Large Scale Systems, Vol. 1, No. 1, February 1980.

[6] C. W. Sanders, E. C. Tacker, T. D. Litton, R. Y. S. Ling, "Specific Structures for Large Scale Eatimation Algorithms Having Information Exchange," IEEE Trans. Auto Control, AC-23, No. 2, 1978.

[7] C. Y. Chong and M. Athans, "On the Periodic Coordination of Linear Stochastic Systems," Automatica, Vol. 12, July 1976.

[8] D. P. Looze and N.R. Sandell, "Decomposition of Linear Decentralized Stochastic Control Problems," ESL-P-288 Proc. 1977 JACC, San Francisco, California June 1977.

[9] S. M. Barta, On Linear Control of Decentralized Stochastic Systems, Ph.D. Thesis, MIT, July 1978.

[10] S. M. Barta and N. R. Sandell, "Certainty Equivalent Solutions of Quadratic Team Problems by a Decentralized Innovations Approach," ESL P-799, MIT, Cambridge, MA, February 1978, Submitted to IEEE Trans. Auto Control.

[11] A. V. Balakrishnan, Applied Functional Analysis, Springer-Verlag, New York 1976.

[12] R. Radner, "Team Decision Problems," Annals of Mathematical Statistics, Vol. 33, 1962.

[13] D. G. Luenberger, Optimization by Vector Spare Methods, John Wiley, New York, 1968.

[14] H. Van Trees, Detection, Estimation and Modulation Theory, John Wiley, 1968.

TECCNET: A Software Testbed for Use in C3 System Research[1]

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Abstract. TECCNET (Testbed for Evaluating Command and Control NETworks) is a small, expandable software system created to support C3 system research. It has been designed to facilitate the modeling and analysis of the complex interactions between the distributed command and control network elements, the algorithms and procedures that characterize the information flow networks of which these elements are a part, and the environment within which they must function. The TECCNET system provides a software laboratory, with a flexible interactive structure, in which basic system research functions are performed. These functions include: the on-line description of the problem of interest and the definition of a model to be studied, the generation of an appropriate scenario, and the execution of the desired experiment. A progression of experiments using TECCNET has been planned to serve a dual purpose. For the near term, the focus will be on the areas of distributed information network management and decentralized estimation. This will allow necessary building blocks to be created, contributing toward the development of an Information Intermediary that is intended to resolve conflicts between the needs of C3 system users and the capabilities of the C3 networks.

INTRODUCTION

The need to provide dynamic limitations on the flow of information in a Command, Control, and Communications (C3) system has become increasingly apparent. Indeed, this need, coupled with the requirement that the C3 systems operate effectively under a variety of adverse conditions, has provided the motivation for much of the recent C3 system research. TECCNET (Testbed for Evaluating Command and Control NETworks) is an experimental software laboratory, designed in response to this need in a way that will support a number of complimentary research activities. Before describing the structure and characteristics of TECCNET, it is appropriate to summarize the point of view taken in creating this research support system.

In this work, the C3 system is visualized as an information flow network. This description encompasses not only the communications systems that transmit data and messages, but also the processing and storage systems that acquire, translate, manipulate, and disseminate information. The performance of this network may be described (at least conceptually) in terms of its ability to deliver, at designated points, the desired information so that upon arrival, it is timely, accurate, complete, and easy to use.

The underlying C3 system problem that motivated the design of TECCNET is extremely complex. The system elements comprising the information flow network are highly distributed, have diverse physical characteristics, and are often governed by ill-defined operational constraints and procedures. The technologies that affect the elements of this system are changing rapidly; advances in electronic weaponry, sensors, and computers, for example, combine with changes in the way information is used by the commander to increase both the flow of information in the network and the time pressures associated with its delivery.

Even under somewhat benign conditions, the task of supporting this flow of information is a formidable one. However, when the tactical situation intensifies, the load on the system increases substantially, just when the external stress on the network induced by a hostile atmosphere is at its peak. Competition for the same resources to move, process, store, and display information also intensifies -- frequently with disastrous results (i.e., excessive message delays, system and user information overloads, etc.). Thus, the C3 system, viewed in terms of how well it provides the

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information support expected by the decision-maker, is perceived to degrade exactly when it is most important that it operate well: when battle information is flowing and the time available for decision making is short. It follows then that there is need to modify the information flow to match it in real-time to the facilities and the time available for processing.

The research problems formulated from the preceding statements share a common premise: in order to develop techniques for controlling the flow of information effectively in a C3 context, one must be able to express and exploit the relationships between the activities of the user and the conditions of the network. This premise is evident in research efforts addressing all levels of the information flow system -- the control of the underlying communication network, the way information is generated and used, and ultimately the interface between the human users and the systems (Ducot, 1980 and 1982). As a result, one of the first goals in the development of TECCNET was to encourage the integration, within a common framework, of the relevant ideas drawn from diverse research areas (i.e., distributed data base, sensor and network management, information processing and presentation, etc.).

Along with the common premise indicated above, a common concern has been expressed over the potential cost (both in terms of system and user resources) of implementing proposed information flow control schemes. Hence the second hope in creating the TECCNET system: that, through experimentation, a better understanding could be developed of the complex interactions between the distributed command and control network elements, the various models, algorithms and procedures that characterize the information flow, the needs of the users of the C3 systems, and the environment within which these systems must function.

Ultimately, however, the objective in creating TECCNET is to promote the development of new models for representing the C3 information processes and new concepts for dealing with the resulting information flow. An important goal, therefore, is to provide the type of environment that will foster a broad range of research activities and will facilitate the testing of proposed algorithms and procedures. In the next section, the design and characteristics of the TECCNET system are described briefly, followed by a discussion of the initial plans for its use as a research tool.

TECCNET: THE SYSTEM IN OUTLINE

In order that TECCNET meet the goals outlined above, a number of design objectives were established. Based on these objectives, a skeleton system was implemented at MIT beginning in 1981. Development of the system continues within this same framework:

1) The testbed should be SMALL, with a controlled plan for expansion, so that it will remain a manageable tool for project research.

2) The software should be structured so that it can operate in a multi-user environment and meet the needs of users with different levels of software and system expertise. Moreover, the system should be interactive and provide considerable on-line documentation.

3) System interface and support software should be developed to facilitate both modeling and testing activities.

4) Default models and representations of the system should be available in order to reduce the effort required to initiate simple experiments.

5) The modeling tools created for the TECCNET system should make it easy for the user to represent the asynchronous interactions and complex protocols that are characteristic of the models and algorithms to be explored.

The preceding statements encompass a broad range of system capabilities--capabilities outside those customarily associated with software for algorithmic and system research. As a result, the final design of TECCNET and the open-ended plans for its development resemble procedures for the design of computer operating systems (Corbato, Saltzer, and Clingen, 1972) as much as they do those applied to the type simulation system software originally anticipated (Oren, Shub, and Roth, 1980).

This dual nature of the software is apparent in the organization of TECCNET, depicted in Figure 1. Three basic research functions, designated 1) the Model Generator, 2) the Scenario and Input Generator, and 3) the Information Network Simulator, are supported within an interactive structure. A brief description of the funtional elements follows. The reader is referred to a discussion of the software system (Ducot, 1982) for additional detail and sample TECCNET interactions.

The Model Generator

The Model Generator is the first of TECCNET components. It permits the user to specify the modeling environment, and, in some sense, to build the simulation on-line. A view of the C3 information flow network is defined by combining models of the local processing nodes, constraints on the

movement of messages, protocols governing the information flow, and the algorithms for managing the network. In general, these specifications are tightly coupled, bound together by the need for consistency in the modeling assumptions. Sets of simulation specifications, along with descriptions of any built-in assumptions, may be stored in the system as defaults -- defaults which can then be manipulated by the user.

For example, one such set (currently in place within TECCNET) permits analysis at the level of the nodes and links comprising a store and forward data communication network. In this case, the queueing model of the processing elements used is extremely simple. Processing of data packets is assumed to take "zero" time compared to packet queueing and transmission delays. The link buffers at the nodes are not modeled explicitly; their capacities are reflected in an effective link capacity that is a fraction of the physical limit of the line. Moreover, the transmission and receipt of packets are assumed to be perfect processes. Certain characteristics of the message traffic (exclusive of volume) also have been specified as defaults. For simplicity, data packets are assumed to have the same average length, and only one conversation may be active between a pair of nodes at any given time. A first-in-first-out (FIFO) service discipline is used for the treatment of data packets, with preemptive logic for both the control packets and acknowledgements having a higher priority in the system.

The structure and content of the data packets are not given for this simple default model; data comprise background traffic within the network. Control messages, on the other hand, require explicit treatment of both structure and content, as these messages are used as signals to drive the TECCNET algorithms. An initial network management algorithm (an outgrowth of an original distributed routing scheme developed by Gallager (1977)) which utilizes the preceding modeling assumptions, is included as part of this modeling environment.

A library structure has been developed to house modeling specifications and algorithmic building blocks. Descriptive information is associated with each entry in the library in order that the user may select among default models and procedures. Additional specifications can be added to modify an existing modeling environment, or to specify a new one as desired by the user.

Scenario Generator

The Input Generator is the data-intensive component of the TECCNET System in which the user defines the conditions to be simulated. For the sample view of the network indicated above, three steps are required: 1) the specification of the network topology, capacity of the links etc., 2) the association of nodes with particular processing models and descriptions of the traffic between them, and 3) the representation of the environment. As the modeling of the information flow network elements becomes more sophisticated, additional inputs representing different types of decision variables will be developed.

Inputs are solicited from the user at the terminal in free form. These data are organized into permanent files and are catalogued with descriptive comments that later may be displayed on-line. The files may be shared between users, each of whom is given a private working copy that functions as a data base during his input session. Sample sessions, indicating how the data bases are created and manipulated by TECCNET, are presented in (Ducot, 1982).

Scenario inputs (describing the condition of the information flow network) are distinguished from those that define the experiment (such as number of iterations, convergence criteria, cost function parameters, type of statistics to be collected, etc.) and are stored separately. This distinction is best appreciated by the user who attempts to combine "canned" scenarios and model specifications for use in multiple experiments. The scenario building process may occur in small segments at different TECCNET sessions until a complete scenario has been obtained and stored in the system.

Network Simulator

Once a model and scenario of interest have been established, the execution phase of the experiment can be initiated. Discrete event simulation techniques form the basis of the execution software. This permits the integration of many procedure-driven models and the represention of asynchronous operation of the elements of a distributed system.

Three types of events are modeled, designated for purposes of discussion "external", "spontaneous", and "responsive". External events are derived from the environment, and refer to situations arising outside the network. These events are not modeled within the system in detail; they are represented only as time-dependent effects applied to one or more of the capabilities of the system elements.

Spontaneous events simulate actions that are based solely on internal logic operating at the nodes of the information flow network. Thus, these internal events correspond to the decoupled actions of a cooperating member of a distributed system. These events may take many forms depending on the modeling environment that has been specified. The most straightforward

spontaneous event is a scheduling event. An example, drawn from the current distributed communication algorithm, is the command from an individual node signalling the initialization of a routing/flow control update cycle that will change the flow within the network. A slightly more elaborate form of the scheduling event is a conditional one, in which some quantity (observable at the node) is monitored until a threshold is reached, at which time the event is scheduled. Internal clocks (synchronized or unsynchronized) are maintained at the nodes to determine the activation time for the spontaneous events.

Execution of a spontaneous event may initiate a sequence of responsive events. Communication with other nodes in the system is required to generate one or more of the events in the sequence. Since responsive events are triggered only by the receipt of an appropriate control message, they are used to model the various *forms* of <u>cooperative</u> actions among the distributed network elements.

The event generator as currently implemented is only partially interactive. The user is on-line while the simulation is running: he may view the results, request that output at various levels be displayed or suppressed, and decide whether or not to continue the experiment. In the future, the user of TECCNET will be permitted a dual role: that of researcher who is observing the experiment, on the one hand, and that of C3 information network customer who is changing inputs and requests in real time, on the other. This additional capability is reflected in the presence of an interactive node in Figure 1.

Conversational Interface

The Conversational Interface provides the link between the user and the body of the TECCNET system. Communication is interactive, with commands and responses entered and displayed at the user's terminal. The forms of the interaction can be controlled by the user, and the display level ("verbose" or "terse") may be set by him according to his familiarity with the system.

This interface software is basically command driven; a feature which gives a user considerable flexibility in his use of the system. This is a "user active" style generally preferred by the designers of interactive systems, reflecting the fact that experienced users can learn to bypass detailed explanations and move efficiently through the system. Occassionly however, when complex descriptions or order-dependent responses must be solicited from the user, the user active mode is suppressed by the Conversational Interface, and a more restrictive question and answer (or "user passive") format is employed.

Whenever the user message (****USER::) appears, TECCNET is awaiting input from the user. A specially designed command-line interpreter monitors the user's entry to distinguish the following: 1) signals for movement within TECCNET (motion commands), 2) requests for information (help commands) and 3) specific data entries (responses to system prompts and questions).

Motion commands allow the transfer between the basic user activities. For example, the command "model" places the user in a position to define his modeling environment. Unlike motion commands, help commands (which provide on-line documentation and clarification) have no positional properties and may be issued without limit at any time. When a help command is received by TECCNET, the information requested is displayed at the terminal, at which time the user may continue his session as though no interrupt had occurred. If a specific question had been asked of the user, the question is repeated at the end of the help message.

These same help messages can also be used to provide on-line training for the user. This requires that an underlying sequence for command use be evident during the interaction. TECCNET messages contain the suggestion of a next logical command at the end of each response; suggestions which may be used by the user to guide him through the system description. As an example, a partial sequence of commands and responses, used as an introduction to the system, is depicted in Figure 2.

USE OF TECCNET

The preceding presentation of the TECCNET modeling system has provided a brief indication of how the software has been structured to support C3 system research. Since the major goal in the development of TECCNET is to promote the evolution of new approaches to information flow control, the plans for using the TECCNET system as a research tool are of immediate interest. A significant payoff is anticipated if the experiments can be structured into experimental building blocks; each of which contributes a portion of the insight and experience necessary to proceed to successively higher levels of abstraction in viewing the information flow network.

One of the chief beneficiaries of such an approach is expected to be the effort to develop the Information Intermediary (Ducot, 1982) which addresses the information related interactions at the highest system level -- the interface between the user of the information and the C3 network itself. The intent of this Intermediary is to assist the human user of the C3 system; aiding him to reformulate his requests for information and to change his use of the information flow network in

light of network conditions. In order to introduce notions of flow control for information (as opposed to data) into the network, the Intermediary must have access to a specially developed local status model of the system; a model that integrates dynamic network, data base, and user information and requires the flow of control information between network elements. In other words, this model must reflect the interactions between three types of information management procedures: 1)strategies that induce changes in routing and control of data flow, given network parameters, 2)criteria for modifying decisions governing the generation and injection of information into the system, given these same or related status indicators, and 3)changes in approaches to information retrieval, given the behavior of the network.

In considering candidate approaches on the basis of their compatiblility and potential contribution to such a model, three desirable features were identified. The first is the feasibility of representing proposed technique for detecting flow conditions and for exerting necessary control, in a way that can be implemented via a distributed algorithm. The second is the ability to formulate the control actions and decisions to be exercised at the nodes based on limited local information. And third is the possibility of sharing common status information and network parameters among different types of management algorithms.

These characteristics were considered in determining the first step in the TECCNET utilization plan; development of a modeling baseline from which a broad class of techniques for managing the communication network could be studied. The initial algorithm included in the TECCNET system is representative of a procedure (type 1) that induces changes in data flow given network conditions. This approach (extended from the original formulation (Gallager, 1977) by Golestaani (1980)) treats flow control and routing together, leading to a flow control algorithm that is expressed in terms of the following conflicting objectives: to reduce congestion in the network while at the same time minimizing the amount of offered traffic that is rejected by that network. A convex optimization problem is formulated in which short-term average information on network utilization is used to allocate both maximum data rates for user sessions (viewed as source/destination pairs) and the optimum routes through the network for information flowing within it (Gallager and Golestaani, 1980). From the point of view of potential contributions to the model required by the Information Intermediary, the appeal of this initial approach lies in the formulation of the distributed algorithm, the type of marginal delay information communicated, and the structure of priority functions that represent the cost of rejecting flow between individual node pairs.

The second of the TECCNET building blocks presumes the existence of both the real-time status information (of the type described above) and the distributed algorithm by which it is communicated. The experiments being considered as part of this second phase (Ozbek, ongoing), represent the first attempt in the TECCNET framework to associate the criteria for generating and injecting information into the network with the network parameters themselves.

The decision variables are drawn from a formulation of a decentralized estimation problem in which explicit use is made of the fact that communication from sensors to estimators is not instaneous. The normal incentives to obtain high quality estimates by transmitting complete information frequently between nodes, are recognized as being far from optimal. As part of this research effort, a number of tradeoffs dealing with the generation and scheduling of information reporting can be addressed. Of immediate interest are those describing: 1) the frequency of reports (relating raw data reporting frequency, traffic volume, delay, and the use of sensor information) and 2) the quality of reports (frequent compressed reports, partially processed at intermediate nodes, versus the less frequent receipt of nearly raw data.

With the experience gained in creating these two building blocks (type 1 and type 2 procedures), it is hoped that the next stage in the TECCNET utilization plan, the incorporation of information retrieval strategies (type 3), may be initiated in the not too distant future.

CONCLUSIONS

In the preceding sections, the design, and intended use of a new research tool, created especially to support C3 system research, was presented. The potential contributions to a variety of ongoing research activities were considered in the development of the initial version of TECCNET, now operational at MIT. Preliminary experience with the TECCNET software suggests that the design objectives, outlined at the beginning of this paper, are being met. The interactive format and modular structure of the system appear appropriate to the needs of users with different levels of software and system expertise who will be participating in this activity in the future. The modeling tools incorporated in the system provide the capability for representing the asynchronous interactions and complex protocols inherent in the models and algorithms likely to be explored. Development of the system is continuing. Additional default modeling environments will be included to allow the pursuit of several lines of inquiry in parallel, each

of which is expected to contribute a different perspective to the overall development of information flow control techniques. It is anticipated that extensive use of the TECCNET system will lead as a by-product to modifications and improvements in the system. As these enhancements are made, it is hoped that the scope of the information flow modeling activities will continue to broaden.

REFERENCES

Corbato, F.J., J.H. Saltzer, and C.T. Clingen, (1972) "Multics--The First Seven Years", AFIPS Conference Proceedings 40, 1972, SJCC, AFIPS Press, Montvale NJ, pp.571-583.

Ducot, E.R. (1980) "Some Thoughts on Information Flow Control in C3 Systems" Volume 5, Proceedings of the Third MIT/ONR Workshop on Distributed Information and Decision Systems, LIDS-R-1024, Laboratory for Information and Decision Systems, MIT, Cambridge MA, December 1980.

Ducot, E.R. (1982) TECCNET: A Testbed for Evaluating Command and Control NETworks, LIDS-R-1227, Laboratory for Information and Decision Systems, MIT, Cambridge MA, August 1982, 63pp.

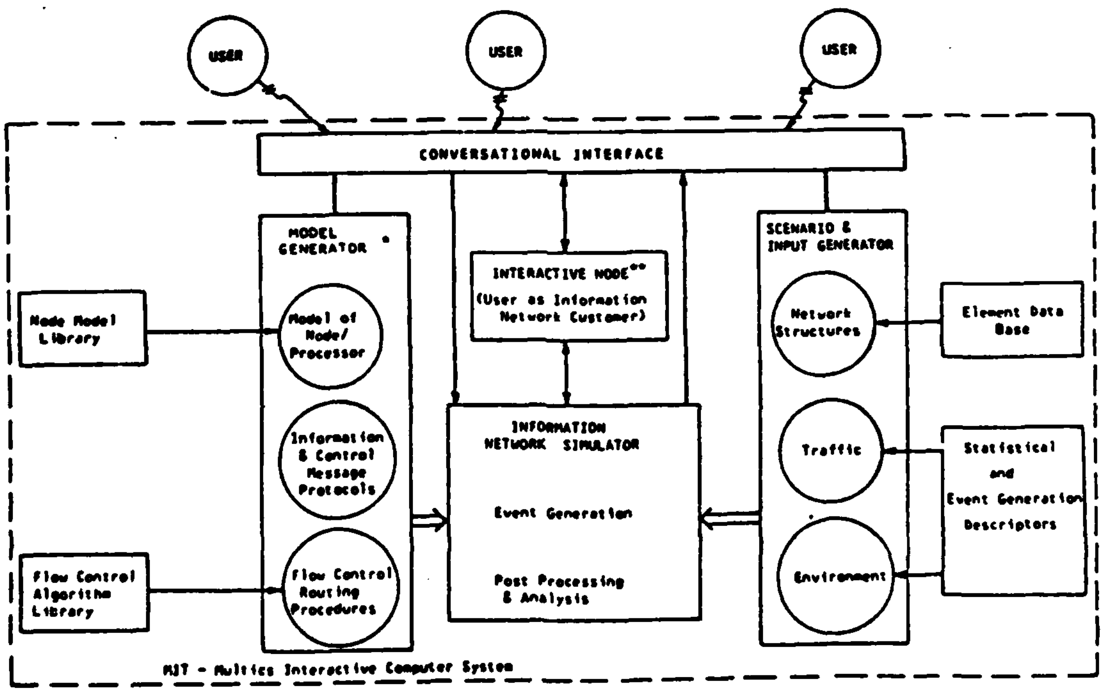
Gallager, R.G., (1977) "A Minimum Delay Routing Algorithm Using Distributed Computation", IEEE Trans. on Communications, Jan. 1977, pp.73-85.

Gallager, R.G. and S.J. Golestaani, (1980) "Flow Control and Routing Algorithms for Data Networks", Proceedings of ICCC'80, Atlanta GA, October 1980.

Golestaani, S.J. A Unified Theory of Flow Control and Routing in Data Communication Networks, PhD Thesis LIDS-TH-963, Laboratory for Information and Decision Systems, MIT, Cambridge, MA January 1980, 92 pages.

Oren, T.I., C.M. Shub , and P.F. Roth, (1980) "Simulation with Discrete Models: A State-of-the-Art View", Volume 2, Proceedings of 1980 Winter Simulation Conference, Orlando FL, Dec. 1980. 258 pages.

Ozbek, A. (ongoing) Unpublished memoranda in preparation for S.M. Thesis, Laboratory for Information and Decision Systems, MIT (scheduled for completion 1983).



*Initial version allows selection of only one process model (simple input/output store & forward node) and one flow control algorithm with baseline message set.

**Planned for future development.

Fig. 1 Structure of TECCNET

```
****TECCNET:

    Welcome to the Information Flow Network Testbed (TECCNET).
 For information on how to use the system, type:
          help
 followed by a carriage return.  Otherwise, on receiving the
 user cue (****USER::) you may type any TECCNET command.

****USER::
help

****TECCNET:
     TECCNET is an interactive testbed which is intended to support
 the analysis of a number of information flow related issues.  It is
 structured to provide user support in three areas:

         1) the specification of his modeling environment, selection
            of local node models, message protocols and flow control
            strategies.  (program segment:  'model')

         2) the generation of simulation input and scenarios that define
            the network / traffic conditions. (program segment: 'scenario')

         3) the on-line execution of the simulation experiment (program
            segment:      'run').

    For information on how to interact with the TECCNET system, the
 inexpert user should type:         use

****USER::
use

****TECCNET:
    You converse with TECCNET by entering commands and responses from your
terminal.  When the user cue (****USER::) appears, it is the indication
that you may begin typing.  To signal the computer that you have completed
your entry you MUST strike the carriage return (CR) key.  The computer will
not respond until you do.
     The commands permit you to move freely within the TECCNET system,
to exercise the various programs, or to receive explanations and
assistance in program selection and data preparation.  Occasionally,
a specific response is required.  In these instances, you will be
prompted from the terminal before the user cue is given.
     Depressing the BREAK or ATTN key at any time will interrupt TECCNET
and return you to a point where you may again enter commands.
   The inexpert user should now type:
                   commands

****USER::
commands

****TECCNET:
    The basic commands for controlling and interacting with the TECCNET
 system are organized into three groups:
         1) movement within the system:
            model: to change modeling environment
            scenario: to create scenario data
            run:    to execute the experiment
            stop:   to leave the system
         2) on-line documentation:
            help:  brief description of TECCNET
            use:    outline of interaction modes
            help model, help scenario, help run: descriptions of
                   the TECCNET submodels
            use model,use scenario, use run: instructions for their use
            help network, help traffic: description of input parameters
         3) user support:
            help error: how to correct errors in typing
            help comment: how to leave comments about the system
            comment: comment mechanism
            terse: request for brief interactive reporting
            verbose: request for complete interactive reporting
```

Fig. 2 Introduction to TECCNET: A Tutorial

END

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